



Research Reports

Multiple Skills Underlie Arithmetic Performance: A Large-Scale Structural Equation Modeling Analysis

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Abstract

Current theoretical approaches point to the importance of several cognitive skills not specific to mathematics for the etiology of mathematics disorders (MD). In the current study, we examined the role of many of these skills, specifically: rapid automatized naming, attention, reading, and visual perception, on mathematics performance among a large group of college students (N = 1,322) with a wide range of arithmetic proficiency. Using factor analysis, we discovered that our data clustered to four latent variables 1) mathematics, 2) perception speed, 3) attention and 4) reading. In subsequent structural equation modeling, we found that the latent variable perception speed had a strong and meaningful effect on mathematics performance. Moreover, sustained attention, independent from the effect of the latent variable perception speed, had a meaningful, direct effect on arithmetic fact retrieval and procedural knowledge. The latent variable reading had a modest effect on mathematics performance. Specifically, reading comprehension, independent from the effect of the latent variable reading, had a meaningful direct effect on mathematics, and particularly on number line knowledge. Attention, tested by the attention network test, had no effect on mathematics, reading or perception speed. These results indicate that multiple factors can affect mathematics performance supporting a heterogeneous approach to mathematics. These results have meaningful implications for the diagnosis and intervention of pure and comorbid learning disorders.

Keywords: perception speed, mathematics performance, pure and comorbid mathematics disability

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Mathematics disorder (MD) is generally defined as poor mathematical abilities related to a specific brain function impairment (Ashkenazi, Rosenberg-Lee, Metcalfe, Swigart, & Menon, 2013; Ashkenazi, Rosenberg-Lee, Tenison, & Menon, 2012). Current theoretical approaches highlight the importance of cognitive skills not unique to mathematics, but rather skills that influence learning processes in general, including reading and attention, for the etiology of MD. For example, Szűcs, Devine, Soltesz, Nobes, and Gabriel (2013) proposed that abnormalities in executive functions, specifically inhibition and visuospatial working memory, are the source of MD, rather than mathematics-specific quantitative understanding. Moreover, there is evidence that these cognitive skills affect individual differences in arithmetic abilities across the spectrum of mathematics proficiency. Szűcs, Devine, Soltesz, Nobes, and Gabriel (2014) demonstrated that individual differences in arithmetic abilities was predicted by phonological processing, verbal knowledge, visuospatial short-term and working memory, spatial ability and executive functioning.

The prevalence rate of MD is between 3-6% among children (Reigosa-Crespo et al., 2012; R. Shalev, Manor, Amir, & Gross-Tsur, 1993; von Aster, Schweiter, & Weinhold Zulauf, 2007; von Aster & Shalev, 2007) while there is less evidence in regard to prevalence rates among adults. However, the prevalence of the comorbidity of MD with other disorders that affect learning (such as reading and attention deficits) vary dramatically between studies, and range between 17-66% (Barbaresi, Katusic, Colligan, Weaver, & Jacobsen, 2005; Dirks, Spyer, van Lieshout, & de Sonneville, 2008; Gross-Tsur, Manor, & Shalev, 1996; Landerl & Moll, 2010; Lewis, Hitch, & Walker, 1994; R. Shalev et al., 1993; von Aster et al., 2007). Even the lowest reported comorbidity rates between MD and reading disorder (RD), RD and Attention Deficit Hyperactivity Disorder (ADHD), as well as MD and ADHD, are higher than expected if the disorders are independent.

These reported prevalence rates suggest that comorbidity between learning disorders is very common. Despite this, most MD research does not take into account the role of comorbidity (Ashkenazi, Black, Abrams, Hoeft, & Menon, 2013). This may be due to the dominance of the core deficit theory, which proposes that MD, RD and ADHD, each originate from unique cognitive mechanisms. Specially, MD as a deficit in quantity representation (Mazzocco, Feigenson, & Halberda, 2011), RD as a phonological deficit (Ramus, 2003), and ADHD as a core deficit in behavioral inhibition (Barkley, 1997).

However, the cognitive processes underlying reading, mathematics and attention are not discrete. Reading and mathematics both involve verbal working memory, cognitive control, and the representation and retrieval of symbolic information (Ashkenazi, Black, et al., 2013). Some mathematics tasks require automatic retrieval of well learned arithmetic facts (e.g., 2 X 3), similar to the need to retrieve semantic information while reading. Moreover, individual differences in arithmetic fact retrieval were found to correlate with phonological awareness, a building block of reading (De Smedt, Taylor, Archibald, & Ansari, 2010). Another fundamental ability in mathematics is an understanding of the Arabic place value system and integration of the components of multidigit numbers. This understanding is important for the number line task in which participants are asked to locate a number on a number line. Successful performance requires verbal understanding of phonological-orthographic relationships related to the Arabic place value system, as well as usage of verbal strategies, thus highlighting the common verbal skills required for both reading and numerical skills (Moeller, Pixner, Kaufmann, & Nuerk, 2009; Peeters, Degrande, Ebersbach, Verschaffel, & Luwel, 2016).

Reading and mathematics both involve executive functions, entangling the relationship between RD, MD and ADHD further due to the central role of executive functions in ADHD (Barkley, 1997). Another related shared risk factor can be processing speed, for example, Peterson et al. (2017) explored the unique and shared risk factors for RD, MD and ADHD and found that slower processing speed was a shared cognitive risk factor. Hence, impairments in any one of the cognitive skills discussed above could contribute to pure MD, RD or ADHD, or any combination of the disorders. Therefore, when examining the skills underlying mathematics performance, it is important to take into account performance in reading, attention and cognitive skills that relate to learning.

Another complication in understanding the etiology of MD is that mathematics itself is heterogeneous. Many different tasks are labeled under the umbrella mathematics, such as counting, arithmetic, and geometry, and each of these tasks can involve different combinations of cognitive skills (Dehaene, 1992; LeFevre et al., 2010). Dehaene's triple code model addressed this issue and proposed three unique codes, verbal, visual and semantic, which are each employed in different kinds of mathematics tasks (Dehaene, Piazza, Pinel, & Cohen,



2003). Similarly, LeFevre et al. (2010) addressed the issue of the heterogeneity of skills underlying mathematics by exploring linguistic, spatial and quantitative pathways to explain performance in a variety of mathematics tasks. They found that linguistic skills in kindergarten predicted performance in seven different mathematics tasks two years later, however, the strength of these relationships varied depending on the task content. For example, the relationship was strongest for numeration, which is a verbal based task, and weakest in magnitude comparison that relies on quantitative understanding. LeFevre's findings, along with Dehaene's triple code model, highlight the variety of cognitive skills that can underlie mathematics performance, and how this can change based on task content.

The Present Study

The goal of the current study was to examine the relationships between mathematics, reading and attention, in order to gain further understanding of the underlying cognitive mechanism of mathematics. To this end we analyzed the task performance of a large dataset of university students (N = 1,322) that came to the Student Support and Diagnostic Centers for Learning Disabilities and ADHD. This large sample included a wide range of arithmetic proficiency. We aimed to examine the underlying skills of mathematics performance, including several number and arithmetic tests: number line estimation, fact retrieval and procedural knowledge.

First, we examined the heterogeneity of the number and arithmetic task performance. Second, we looked at the role of phonological processing, verbal abilities, attention and perception speed in explaining individual differences in numerical and arithmetic abilities. We predicted that linguistic skills would positively relate to all of the mathematics tasks; however, different components of linguistic skills would affect each of the tasks differently. Specifically, phonological processing would relate to fact retrieval (De Smedt et al., 2010), while verbal abilities, such as reading comprehension, would relate to number line estimation (Peterson et al., 2017). In addition, we predicted that processing speed would affect performance in all of the number and arithmetic tasks (Peterson et al., 2017). However, we expected the effect of processing speed would be greater in fast and automatic tasks, such as retrieval of arithmetic facts. Last, attention should contribute to individual differences in performance in all the number and arithmetic tasks (LeFevre et al., 2010).

Method

Participants

The participants were university students who came to the Student Support and Diagnostic Center for Learning Disabilities and ADHD for assessment to receive testing accommodations and benefits. The participants signed a consent form allowing their assessment results to be accessed for future research. Our dataset included 1,322 students that came to 3 test centers in Israel between the years 2007 and 2014. All the participants performed the entire diagnostic battery of the *MATAL* (*MATAL* is an acronym in Hebrew, in English it translates to: Learning Functioning System), a computerized, normed diagnostic tool that is used for assessing learning disabilities for college students in Israel (Shalev, Ben-Simon, Mevorach, Cohen, & Tsal, 2011). Each student that was tested at the Center and completed the entire battery was included in our analysis. Table 1 presents the demographic information for the entire dataset.



Table 1

Demographics of the Dataset (N = 1.322)

| Characteristic | Value |
|----------------------|---|
| Age (in years) | <i>M</i> = 23.19, <i>SD</i> = 2.77, Range = 16-48 |
| Female | 52% |
| Birth country Israel | 85% |
| Hebrew mother tongue | 91% |

Tools

The *MATAL* was developed by Israel's National Institute for Testing and Evaluation. The *MATAL* assesses proficiency in mathematics, reading, writing, and English (as a second language), as well as ADHD. Other than the English tasks, all of the *MATAL* tasks are conducted in Hebrew. The diagnostic battery includes two questionnaires and 20 tasks (the tasks relevant for the current study are described below, for a complete description see: (Shalev et al., 2011). Students complete the entire battery in two sessions that last approximately three hours each with an examiner present, followed by an intake session with a psychologist. The scores for each task in the *MATAL* are standardized and reaction time (RT) measures are converted so that positive higher scores in both accuracy and RT measures are indicative of superior performance. It is important to note that the scores do not distribute normally, rather they have a negative skew resulting in more extreme negative standard scores than is found in normal distributions (Shalev et al., 2011).

Mathematics Tasks

Calculation Automaticity

The goal of the task was to measure retrieval of arithmetic facts. The task included 80 simple arithmetic equations (e.g. $2 \times 3 = 6$) that were presented sequentially on the computer screen, the participant had to answer if the equation was correct or incorrect by keypress. The equations divided equally into addition, subtraction, multiplication and division problems (20 per category). The equations included 1-digit numbers and the possible solutions did not exceed 25. The equations were either correct (2 + 2 = 4, 40% of the trials), incorrect but with an associative distractor (2 + 6 = 12, 30% of the trials), or incorrect (3 * 4 = 10, 30% of the trials).

For the practice trials, an equation would appear for 10 seconds, while test trials were presented for 4 seconds, or until the participant selected an answer. Percent of correct answers and average reaction time (RT) per trial were measured.

Procedural Knowledge

The task was designed to measure procedural knowledge. It comprised of 96 equations that were presented sequentially on the computer screen, and participants needed to ascertain if the equation was correct or incorrect by keypress. The equations included numbers that ranged from 1-4 integers. All the equations required logarithmic, simple calculations. The equations divided equally into addition, subtraction, multiplication and division, and evenly into correct and incorrect solutions within each category (i.e. 12 addition problems with the correct solution). For the practice trials, an equation would appear for 10 seconds, while test trials were



presented for 6 seconds or until the participant selected an answer. The entire task was about 10 minutes. Percent of correct answers and average RT per trial were measured.

Number Line Knowledge

This task measured understanding of the mental number line. It included 58 number lines that were presented sequentially on the computer screen. For each trial, different values appeared at the anchors of the number line, in black font, below the line (see Figure 1). Unlike the classic number line task that has one target, the *MATAL* version has two target points marked with the same value and the participant had to ascertain which of the target points was marked correctly. The distance between the target points was 20% or 40% of the length of the whole line. The number lines included: natural numbers, fractions and negative numbers. Each practice trial was presented for 20 seconds and test trials for 10 seconds or until the participant selected an answer. Percent of correct answers and average RT per trial were measured.

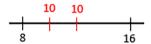


Figure 1. Example of trial from the Number Line Knowledge Task.

Reading Tasks

Reading Text

A 132-word text appeared on the computer screen. The participant was instructed to read the text aloud as accurately and quickly as possible without follow up questions. The results were voice-recorded via the computer and the examiner noted errors. Accuracy was measured as the percentage of words read correctly and pace as number of words per minute.

Pseudoword Reading (Phonological Encoding)

The task included 25 pseudowords that were presented sequentially on the computer screen. The words were displayed in pointed font. Hebrew letters are similar to consonants in English, while points are analogous to English vowels. After reading acquisition, Hebrew is easily read without points, however, they are required in order to read new words or pseudowords. All the words were made up of 2 or 3 syllables and followed grammar rules. None of the words looked or sounded like actual Hebrew words. The only way the words could be read was by grapheme-phoneme encoding. The participant was instructed to read the words aloud as accurately and quickly as possible. The participants' results were voice recorded and the examiner marked errors on line. The measures taken were: accuracy, measured as the percentage of words read correctly, and RT, average time per item.

Encoding Pseudowords (Forced Choice)

This task involved both grapheme-phoneme encoding and lexical retrieval. It included 40 pairs of pseudowords that were presented sequentially on the screen, similar to those from the previous task, however, they could sound like a word in Hebrew. The pairs of words differed by one consonant or one vowel (e.g. luv- buv). The participant had to decide by keypress which word sounded like a real word in Hebrew. Each pair was presented until the participant answered (maximum presentation time was 10 seconds) and the entire task was



approximately 8 minutes. Accuracy, measured as percentage of correct answers, and average RT per trial, were measured.

Phoneme Omission

This task measured phonological awareness. The participant heard recordings of 24 pseudowords via the computer. Each trial, the participant heard a pseudoword and was instructed to repeat it aloud. The pseudoword was played again and the participant was instructed to repeat the word without a specific sound. The maximum time allotted each trial was 10 seconds, and the entire task was approximately 5 minutes. The measures were accuracy, measured as percentage of correct answers, and average RT per trial.

Reading Comprehension

The task included three texts, each text was followed by 9-11 questions (there was a total of 30 questions). The texts were ordered in level of difficulty, ranging from easy to difficult. The task was presented on the computer screen, and the students could click to go back and forth between the text and the questions. The questions were multiple choice with four possible answers per question. The questions included: basic understanding and fact finding, relationships between different parts of the text, and text interpretation. Performance was measured as the percent of correct answers from all of the questions.

Attention Tasks

Continuous Performance Task (CPT)

The CPT measured sustained attention and impulsivity based on the principles of the attention tasks created by Tsal, Shalev, and Mevorach (2005). For the task, one stimulus at a time was presented sequentially on the computer screen. The stimuli differed on two dimensions: shape and color. The participant was instructed to push a key when the target stimulus appeared (e.g. green triangle), and not to respond to the other stimuli. The target stimulus appeared 30% of the trials. The distractor stimuli could differ in color but not shape (17.5% of the trials), in shape but not color (17.5%) or on both dimensions (35%). Each stimulus appeared for 100 milliseconds, the entire task lasted about 20 minutes. Omission errors was the percentage of target stimuli a participant missed and commission errors was the percentage of false alarms (i.e. participant responded in the absence of the target stimulus), which was measured separately for the first and second half of the task.

Attention Network Test (ANT)

The task used in the *MATAL* was based on the original ANT developed by Fan, McCandliss, Sommer, Raz, and Posner (2002) that measured the three components of attention: alerting of attention, orienting of attention and executive attention. For each trial, a horizontal row of five lines appeared on the screen either above or below a fixation point. The center line was an arrow that could point left or right. The participant's task was to ascertain if the middle line was pointing left or right. However, the lines on either side of the center arrow could be either neutral, congruent or incongruent.

In addition, the trials had varying types of cues. Some trials had no cue, others had center cues in the same location as the fixation point, double cues that appeared above and below simultaneously, or the spatial cue that appeared above or below the fixation point. The spatial cue could appear in the same location as the target stimulus, or the opposite location as the target stimulus. The target stimulus was presented for 1.7 seconds, while the whole trial lasted 4 seconds. The task lasted approximately 30 minutes.



There were several measures taken from this task. 1) Executive attention, the average RT for incongruent trials minus the average RT for congruent trials; 2) Alerting/arousal, average RT for un-cued trials minus the average RT for double cue trials; 3) Orienting of attention, the average RT of trials with central cues minus the average RT for spatial cue trials.

Speed of Processing Tasks

Rapid Automatized Naming (RAN)

The task was based on (Denckla & Rudel, 1976) and included three types of stimuli: objects, letters and numbers, each stimulus type was presented in its own block. Each series was displayed in a 10x5 matrix, totaling 50 stimuli per set. The participant was instructed to name the stimuli aloud as quickly as possible. Each stimulus set appeared for a set amount of time. Efficiency scores were measured for each stimulus set separately, as the number of items named within the given time frame.

Visual Perception Task

This task had two conditions:

- 1) Parallel: this measured visual-spatial perception using a spatial frequency discrimination task. In each trial two horizontal gratings appeared simultaneously on the top and bottom halves of the computer screen. The two stimuli differed in spatial frequency. The participants had to ascertain whether the top or bottom was denser by pressing the up or down arrow keys on the keyboard. Each trial the reference grating appeared in either location along with a test grating. The task began with a contrast of 20% between the reference and test stimuli, the contrast adapted to the participant's performance. Specifically, the reference and the test stimuli become more similar after every two correct answers or became less similar after every incorrect answer. The task ended when the participant reached his/her just noticeable difference or 80 trials. The just noticeable difference was the maximum density that a participant could correctly identify (defined by 18 replacements from increase in the density of the test stimuli to increased).
- 2) Sequential: Similar to the other condition. While in the parallel task the two gratings appeared simultaneously, in the sequential task the gratings were presented sequentially and filled the entire computer screen (Ben-Yehudah & Ahissar, 2004).

Results

Correlation Analysis

To eliminate multicollinearity in future analysis, we examined the correlations between all of the tasks of the MATAL. The MATAL battery included 20 tasks, and most of the tasks had measurements of both RT and accuracy, resulting in approximately 40 measurements. Accordingly, the first step was reducing the number of independent variables; tasks with a correlation greater than .6 between RT and accuracy were averaged to create an efficiency score. Then, high correlations (greater than .6), between tasks that require similar, theoretical cognitive mechanisms were combined. The cutoff of r = .6 was chosen ad hoc to aggregate the data, and was not set by a statistical method or inference. This 1) combined 4 measures of 2 mathematical tests, 2) combined 4 reading tests to 2 pairs and 3) combined RT and accuracy measures of 3 reading tasks.



For a summary of all the correlations see Supplementary Materials. For the mathematics tasks, procedural knowledge and calculation automaticity, accuracy and RT highly correlated for each of the tasks (procedural knowledge- the correlation between RT and accuracy was r(1320) = .71, p < .01, calculation automaticity- the correlation between RT and accuracy was: r(1320) = .60, p < .01), we calculated efficiency scores for each task, by averaging accuracy and RT (all the scores were standardized as discussed above). In sum, we averaged the scores of: 1) pseudoword reading and identification, and 2) RAN letters and numbers. In addition, we calculated efficiency scores for 1) pseudoword reading and identification and 2) phonological omission. The correlations between all the observed variables after the above corrections are presented in Table 2.

Table 2

Relationships Between MATAL Tasks After Recalculating Scores

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|----|---------|---------|---------|--------|--------|--------|-------------------|-------------------|---------|---------|------------------|---------|---------|--------|--------|
| 1 | - | | | | | | | | | | | | | | |
| 2 | .426**a | - | | | | | | | | | | | | | |
| 3 | .558**a | .407**a | - | | | | | | | | | | | | |
| 4 | .316**a | .385**a | .295**a | - | | | | | | | | | | | |
| 5 | .302** | .291** | .265** | .310** | - | | | | | | | | | | |
| 6 | .349** | .299** | .278** | .298** | .592** | | | | | | | | | | |
| 7 | .273** | .101** | .227** | .322** | .413** | .403** | - | | | | | | | | |
| 8 | .178** | 0.095 | .134** | .204** | .304** | .249** | .440**b | - | | | | | | | |
| 9 | .037 | .005 | .000 | .029 | .031 | .029 | 020 ^b | 001 ^b | - | | | | | | |
| 10 | .024 | 011 | .011 | 010 | .015 | .026 | .006 ^b | 002 ^b | .498**b | - | | | | | |
| 11 | 023 | 019 | 033 | 005 | .019 | .007 | 005 ^b | .008 ^b | .321**b | .360**b | - | | | | |
| 12 | .333** | .219** | .309** | .389** | .380** | .354** | .487** | .329** | 019 | 002 | 025 ^c | - | | | |
| 13 | .313** | .163** | .327** | .355** | .291** | .274** | .264** | .171** | .013 | .036 | 018 ^c | .718**c | - | | |
| 14 | .146** | .108** | .281** | .300** | .223** | .186** | .283** | .257** | .007 | .016 | 034 ^c | .504**c | .587**c | - | |
| 15 | .269** | .136** | .304** | .283** | .386** | .365** | .492** | .330** | 017 | 006 | .005 | .405** | .366** | .348** | - |
| 16 | .290** | .193** | .318** | .336** | .422** | .396** | .526** | .417** | .016 | .032 | .017 | .446** | .398** | .400** | .685** |

Note. 1. pseudoword reading and identification efficiency score; 2. Text reading efficiency score; 3. Phonological omission efficiency score; 4. Reading comprehension efficiency score; 5. RAN objects accuracy; 6. RAN number letter accuracy; 7. CPT omission; 8. CPT commission; 9. ANT orienting; 10. ANT alertness; 11. ANT executive; 12. Calculation automaticity efficiency score; 13. Procedural knowledge efficiency score; 14. Number line knowledge accuracy; 15. Visual processing parallel; 16. Visual processing serial.

aCorrelations between reading tasks. Correlations between attention tasks.

As can be seen in Table 2, all the reading tasks had medium to high correlations ranging from r(1320) = .34, p < .001 to r(1320) = .60, p < .001. The mathematics tasks had high correlations as well, ranging from r(1320) = .50, p < .001 to r(1320) = .72, p < .001. Last, the perception speed tasks RAN and visual processing significantly correlated r(1320) = .27, p < .001 and r(1320) = .68, p < .001.

For the attention tasks, there were correlations between CPT omission and commission, r(1320) = .44, p < .001 and between the measures of the ANT task ranging from r(1320) = .32, p < .001 to r(1320) = .50, p < .001. However, there were no significant correlations between CPT and ANT ranging from r(1320) = .001, p = .97 to r(1320) = .008, p = .77. Rather, CPT significantly correlated to perception speed, RAN and visual processing,



^{**}Correlation is significant at the Bonferroni adjusted alpha p = .000417.

ranging from r(1320) = .27, p < .001 to r(1320) = .53, p < .001. The ANT task did not significantly correlate with any of the other variables.

Data Reduction Analysis

Following these modifications, we performed a principal component analysis with promax Kaiser Normalization rotation on the entire data set (N = 1,322) extracted from the *MATAL* to examine how performance in the various tasks related to one another. We found 4 factors with eigenvalues greater than 1, which explained 49.3% of the total variance (see Table 3).

Table 3

Principal Component Analysis

| | Rotate component matrix promax with Kaiser Normalization | | | | | | | | | |
|---------------------------------------|--|-------------------|----------------------|------------------------|--|--|--|--|--|--|
| | Factor 1: Cognitive processes | Factor 2: Math | Factor 3: Reading | Factor 4: Attention | | | | | | |
| Measure | (18.9%) | (16.3%) | (7.7%) | (6.3%) | | | | | | |
| CPT omission | 0.81 | | | | | | | | | |
| VP serial | 0.77 | | | | | | | | | |
| VP parallel | 0.71 | | | | | | | | | |
| CPT commission | 0.63 | | | | | | | | | |
| RAN objects | 0.52 | | | | | | | | | |
| RAN letter/number | 0.47 | | | | | | | | | |
| Procedural knowledge | | 1.11 | | | | | | | | |
| Calculation automaticity | | 0.62 | | | | | | | | |
| Number line knowledge | | 0.54 | | | | | | | | |
| Pseudoword reading and identification | | | 0.78 | | | | | | | |
| Phoneme omission | | | 0.72 | | | | | | | |
| Text reading | | | 0.69 | | | | | | | |
| Reading comprehension | | | 0.38 | | | | | | | |
| Alerting | | | | 0.74 | | | | | | |
| Orienting of attention | | | | 0.67 | | | | | | |
| Executive attention | | | | 0.49 | | | | | | |

Note. Proportion of variance is accounted for by each factor and loading of each measure on factor.

We termed the first factor "perception speed" that included the following tasks: RAN (object and number/letter), CPT (omission and commission), and visual perception (serial and parallel) because all of the tasks required fast responses to a visual stimulus. The second factor we titled "mathematics" that included efficiency scores for calculation automaticity and procedural knowledge, and number line knowledge. The third factor we titled "reading" that included reading tasks: pseudoword reading and identification, reading comprehension, phoneme omission and text reading. The last factor we titled "attention" that included the 3 measures of the ANT task: alerting, executive attention and orienting of attention.

Structural Equation Modeling (SEM) Analysis

Model 1 of the SEM was designed based on the results of the principal component analysis. This was a statistical-fit led model, data driven from the previous analysis. We tested an independent three-factor model:



perception speed, reading, and attention as independent influences of mathematics ability (see Figure 2; standardized values). Both reading and perception speed predicted mathematics (β = 0.26 and β = 0.75, p < .001) respectively. However, attention did not (β = -0.006). The model fit was poor $\chi^2(98)$ = 1201, p < .001 (χ^2/df = 12.25; CFI = 0.86; RMSEA = 0.09). Accordingly, we modified the model according to the suggestion of the AMOS program. 1) We added a direct effect between CPT omission and mathematics automaticity and another direct effect between CPT omission and procedural knowledge, which improved the model, χ^2 change = 173.3, p < .001. 2) We added a direct path between reading comprehension and the latent variable mathematics, and reading comprehension and number line knowledge, which improved the model, χ^2 change = 69.3, p < .001. 3) We added a direct path between RAN letter/number and text reading, which improved the model, χ^2 change = 28, p < .001. 4) We deleted the observed variable RAN objects and serial visual processing due to high correlations with other observed variables (RAN number/letter and parallel visual processing respectively). 5) We add a direct path between the latent variable perception speed and text reading, which improved the model, (χ^2 change = 85, p < .001) (see Figure 3; standardized values). In the new model, both reading and perception speed predicted mathematics (β = 0.19 and β = 0.79, p < .01 and p < .01 respectively. The model fit significantly improved χ^2 (66) = 347, p < .001 (χ^2 /df = 5.26; CFI = 0.95; RMSEA = 0.06).

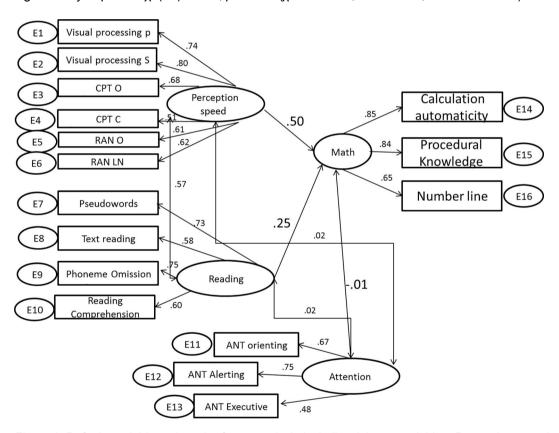


Figure 2. Default model based on the factor analysis including 3 latent variables: Perception speed, reading and attention (ANT) predict mathematics abilities.

Visual processing P = visual processing parallel; Visual processing S = visual processing serial; CPT O = continuous performance task omission; CPT C = continuous performance task commission; RAN O = rapid automatized naming objects; RAN LN = rapid automatized naming letter/number; ANT orienting = attention network test orientation; ANT Alerting = attention network test alertness; ANT executive = attention network test executive.

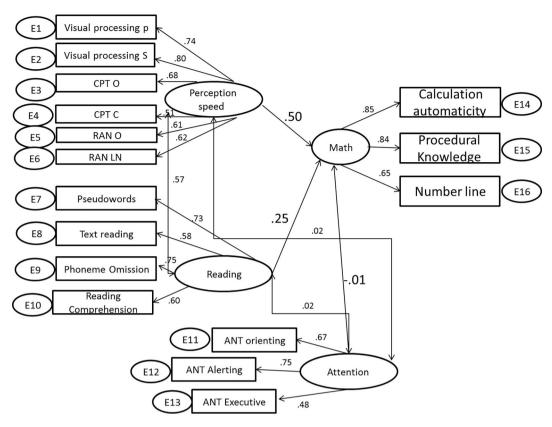


Figure 3. Final model based on the factor analysis including 2 latent variables: perception speed and reading predict mathematics abilities. Attention (ANT) did not have direct role in explaining mathematics abilities in the current model.

Visual processing P = visual processing parallel; CPT O = continuous performance task omission; CPT C = continuous performance task commission; RAN LN = rapid automatized naming letter/number; ANT orienting = attention network test orientation; ANT Alerting = attention network test alertness; ANT executive = attention network test executive.

The Role of Attention

The previous model indicated that attention tested by ANT did not play a significant role in explaining mathematics task performance. However, a possible explanation is that attention was predictive of perception speed and reading that in turn predicted mathematics, Figure 4 presents this kind of model. To test this hypothesis, we constructed an alternative model using SEM. The result of the model indicated that attention did not predict reading, perception speed or mathematics (β = 0.002 and β = 0.011, β = 0.019, N.S. for all values). The model fit was poor (χ^2 /df = 7.88; CFI = 0.922; RMSEA = 0.072). Moreover, omitting the direct connection between attention and reading, attention and perception speed, and attention and mathematics did not significantly reduce the χ^2 (χ^2 change = .008, ρ = .92; χ^2 change = 0.154, ρ = .69; χ^2 change = .062, ρ = .84, respectively). Therefore, the proposed model, with the indirect effect of attention on mathematics ability, was not supported by the results.



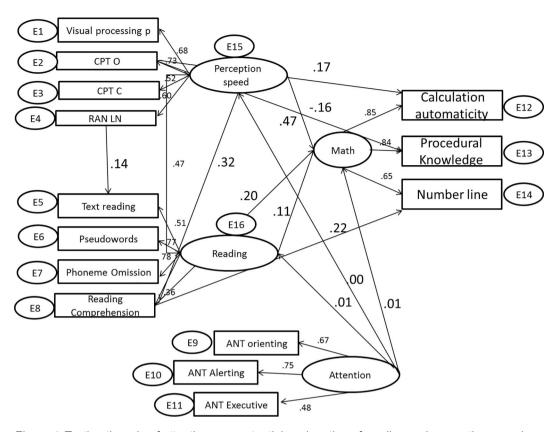


Figure 4. Testing the role of attention as a potential explanation of reading and perception speed.

Visual processing P = visual processing parallel. CPT O = continuous performance task omission. CPT C = continuous performance task commission. RAN LN = rapid automatized naming letter numbers. ANT orienting = attention network test orientation. ANT Alerting = attention network test alertness. ANT executive = attention network test executive.

Discussion

The goal of the current study was to examine how performance in a comprehensive learning disorder assessment relates to mathematics performance, in a large group of adults that represented a wide range of arithmetic proficiency. First, in the factor analysis, we found that the mathematics subtests of the *MATAL* all loaded on the same factor, suggesting that all mathematical tasks share some numerical entities, which leads to gravitational pull between them. However, our SEM demonstrated that despite the shared variance across the mathematics tasks, procedural knowledge and calculation automaticity were strongly related, and influenced directly by CPT omission, while the number line task was unique, and was influenced by reading comprehension.

Second, perception speed and reading, but not attention, shared unique variance with mathematics ability. We discovered that performance in the latent variable perception speed strongly predicted the latent variable mathematics. Moreover, the latent variable reading also had a direct influence on the latent variable mathematics. Importantly, the task reading comprehension had a strong and direct effect. We found no direct or indirect influence of the latent variable attention, tested by the ANT, on the latent variable mathematics, reading or perception speed.



Multiple Arithmetic Abilities

Different types of arithmetic and number tasks can require different combinations of cognitive skills. Dehaene's triple code model postulated that different skills are recruited based on the task demands (Dehaene et al., 2003). The verbal code is activated during well-rehearsed tasks such as retrieval of arithmetic facts, while the semantic code is activated during tasks that require representation of numbers on the mental number line (Dehaene et al., 2003). The semantic code is unique to mathematics, while the verbal code is shared with reading.

According to Dehaene et al.'s model (2003) number representation in the semantic code is based on the mental number line. Hence, classic approaches (see Siegler & Opfer, 2003) viewed the number line task as a measurement of pure numerical estimation. However, newer studies explored fresh perspectives underlying the number line task and have highlighted the importance of place value integration and lexical understanding in the solution of the number line task (e.g., Barth & Paladino (2011). Moeller et al. (2009) proposed that young children have two separate representations for 1-digit and 2-digit numbers rather than a single holistic representation, which influences the ability to correctly solve the number line task. Later, Barth and Paladino (2011) argued that number line tasks should be treated as proportion estimation, suggesting that participants use reference points during number line task solution. With maturation and schooling, children learn to use the reference points, which subsequently improves their performance in the task.

Both of these studies emphasize that improvement in the number line task is based on verbal knowledge of the symbolic number system, which includes understanding the relationship between hundreds, decades and units and of place value that is based on lexical rules. Accordingly, a cross-cultural study found that Italian speaking 1st graders had higher accuracy in number line estimation task performance compared to German speaking 1st graders. The difference between the two languages is the unique inversion property of German multi-digit numbers (e.g., 48 \rightarrow "eight-and-forty") (Helmreich et al., 2011). This result demonstrated that number line estimation task performance can be influenced by the phonological-orthographic relation of language. Hence, phonological awareness that is related to phonological-orthographic relation should uniquely predicted number line task performance. However, in the current study we found that reading comprehension, not phonological awareness, uniquely predicted number line task performance. Please note that Peeters et al. (2016) suggested that number line task solution requires verbal strategies such as using the verbally mediated benchmark, skills which likely share some variance with reading comprehension.

Performance in the arithmetic tasks, procedural knowledge and calculation automaticity, highly correlated and were directly affected by sustained attention. CPT omission directly related to sustained attention, which is typically measured during long and boring tasks. Procedural knowledge and calculation automaticity are both easy and automatic tasks for adults, in comparison to children who calculate the answer each trial. Due to the involvement of retrieval and automatic nature of the tasks for adults, beyond calculation they also require sustained attention. Each trial the participant needs to attend to the arithmetic symbols and place value of the numbers, requiring constant vigilance for the duration of the task. These findings support the heterogeneous perspective (Dehaene et al., 2003; LeFevre et al., 2010): the different mathematics tasks measured in the current study had different relationships with underlying cognitive skills contingent on the task demands. Procedural knowledge and calculation automaticity related to sustained attention, while number line task performance had a unique relationship with reading comprehension.



Two Different Influences of Reading on Mathematics Abilities, The Role of Verbal Comprehension

We found that reading and phonological awareness predicted arithmetic abilities. The factor analysis results grouped reading comprehension, text reading and phonological awareness under one latent variable we named "reading". However, the SEM results clearly differentiated between the effect of the latent variable reading, and variance related uniquely to reading comprehension. The contribution of reading comprehension was slightly larger than the contribution of the latent variable reading overall.

Peterson et al. (2017) examined the cognitive skills underlying reading using factor analysis and found two independent latent variables they labeled "phonological awareness" and "verbal comprehension". While verbal comprehension was a shared risk factor for reading and mathematics, phonological awareness was a unique risk factor for reading deficits. One possible explanation is that verbal comprehension requires verbal working memory that is shared between reading and mathematics (Pennington, 2006). Here, we demonstrated that understanding of verbal information also had a direct effect on mathematics abilities. However, contrary to Peterson et al.'s (2017) results, we also found a direct effect of the latent variable reading, which included phonological awareness, on mathematics abilities.

The present study is not the first to suggest that phonological awareness can explain individual differences in mathematics performance, several studies point to the critical role of phonological awareness on mathematics abilities in children (Durand, Hulme, Larkin, & Snowling, 2005; Fuchs et al., 2006; Rasmussen & Bisanz, 2005). There is evidence that symbolic representation of number is language-based and arithmetic facts are stored verbally in long-term memory (Simmons & Singleton, 2008). The calculation automaticity and procedural knowledge tasks from the *MATAL* are heavily language-based because they rely on retrieval and mathematic rules, respectively. Moreover, De Smedt et al. (2010) found a relationship between the ability to retrieve small arithmetic facts and phonological abilities, which they did not find for the calculation of large operations (De Smedt et al., 2010). In addition, the connection between phonological abilities and retrieval of arithmetic facts can be based on a shared brain mechanism, the left angular gyrus (Dehaene et al., 2003). Our results suggest that impaired phonological awareness can partially explain the comorbidity between MD and RD, and in fact Slot, van Viersen, De Bree, and Kroesbergen (2016) found that impaired phonological awareness was a shared risk factor for pure MD, pure RD and partially explained the comorbidity between MD and RD.

The Role of Perception Speed in Mathematics Abilities

The latent variable perception speed explained a large proportion of the variability in mathematics abilities. The latent variable perception speed was composed of tasks that required rapid responses to visual stimuli. One of the main tasks that test fast response is RAN. Wilson et al. (2015) examined RAN performance among RD and MD adults and found similar deficits in both groups. Two recent studies found RAN deficits in children with both MD and RD (Donker, Kroesbergen, Slot, Van Viersen, & De Bree, 2016; Slot et al., 2016). Moreover, other studies found that RAN deficit was a shared risk factor between ADHD and reading (McGrath et al., 2011; Shanahan et al., 2006). Last, naming speed was found to predict mathematics abilities among typically developing children (Swanson & Kim, 2007). Hence, perception speed, as tested by RAN and other cognitive tests (including visual processing and CPT) can be a potential source of vulnerability for the comorbidity between RD, MD and ADHD.



It is important to recognize that others have found that poor RAN performance is a unique characteristic of RD and not MD (Moll, Gobel, Gooch, Landerl, & Snowling, 2016). Partially in line with this view, we found that part of the shared variance of RAN explained in the latent factor "perception speed" had a direct effect on text reading. Hence, it has a potentially larger role explaining reading deficits over mathematics deficits.

The Role of Sustained Attention in Mathematics

Studies investigating the role of sustained attention in explaining individual differences in mathematics abilities is inconsistent, some studies demonstrated a relationship (Ashkenazi, Golan, & Silverman, 2014; Steele, Karmiloff-Smith, Cornish, & Scerif, 2012), while others have not (Szűcs et al., 2014). Steele et al. (2012) examined the explanatory role of attention for literacy and numeracy among 6-year-old children over the course of a year. They found that sustained attention and selective attention predicted outcomes in numeracy, but not literacy. We found in a previous study that adults with superior sustained attention had higher accuracy in addition operations (Ashkenazi et al., 2014). However, Szűcs et al. (2014) found that sustained attention did not play a significant, direct role in explaining individual differences in mathematics performance for 8-year-old children. The result of the present study can explain this inconsistency; we found that sustained attention, tested by CPT omission rate, directly explained variability in calculation tasks but not the number line task. Hence, the relationship between mathematics and sustained attention may be contingent on the tasks used to measure these abilities.

The ANT Task Did not Explain Learning Abilities

There are some indications that participants with learning disorders have deficits in some of the attentional networks tested by the ANT. Specifically, ADHD participants have presented poor executive attention and orienting of attention (Mogg et al., 2015), while deficits in executive attention have also been found in RD (Bednarek et al., 2004) and deficits in executive attention and alerting network in MD (Askenazi & Henik, 2010). Please note that the above studies tested limited sample sizes (*n* range between 14 - to 67 in each group). However, in our large sample, attention tested by a version of the ANT did not have an explanatory role in perception speed, reading or mathematics, leading to the conclusion that we should look at the presumed effect of attentional factor (tested by the ANT) on learning abilities with extra caution.

Limitations

Our analysis was based on a population that came for learning disability assessment, and therefore, inferences from our findings can be drawn only to a limited extent for the whole population. Moreover, the tasks examined in the present study were drawn directly from the *MATAL*. Hence, we could not design the tasks to evaluate other important variables in numerical cognition literature, such as logarithmic representation in the number line task, change the forced-choice response method for procedural knowledge or include the symbolic and non-symbolic comparison tasks (Ansari, Price, & Holloway, 2010). In addition, there are other relevant tasks that are not included in the *MATAL* battery. Current research proposes that domain general processes effect arithmetic processing including working memory (mostly visuospatial working memory) and executive function (such as inhibition) (Ashkenazi, Rosenberg-Lee, et al., 2013; Szűcs, Devine, Soltesz, Nobes, & Gabriel, 2013). Working memory and executive function tasks needs to be included in future similar studies. Last, the current study points to critical influences of multiple cognitive skills on arithmetic processing. However, the need to understand these underlying processes is still open and should be addressed in future studies.



Conclusions

We examined performance in mathematics, reading, attention and cognitive tasks in a large data set of college students (N = 1,322) to shed light on how a variety of cognitive skills can relate to performance in different mathematics tasks. We discovered that many of these skills played a significant explanatory role for adult's mathematic abilities. Specifically, perception speed (which included: visual processing, RAN, CPT) was the latent factor that explained the most variance in arithmetic abilities. Moreover, reading (which included: pseudoword reading and identification, phonological omission, text reading and reading comprehension) also explained variability in mathematics performance. Interestingly, reading comprehension on its own had a meaningful, direct effect on mathematics performance.

We found different relationships between cognitive skills and the different mathematics tasks. Retrieval of arithmetic facts and procedural knowledge were influenced directly by sustained attention, while number line knowledge was directly affected by reading comprehension, demonstrating the heterogeneity of arithmetic and number tasks. These results have meaningful outcomes for the diagnosis and intervention of pure and comorbid learning disorders.

Notes

i) Please note that for theoretical reasons we did not average 1) RAN numbers, letters and objects. 2) Visual processing serial and parallel. 3) Procedural knowledge and calculation automaticity. 4) Phonological omission and pseudoword reading and identification even though the correlations between each of these pairs was > .6.

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Competing Interests

The authors declare no competing financial interests.

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