מרכז ארצי לבחינות ולהערכה (ע"ר)
NATIONAL INSTITUTE FOR TESTING & EVALUATION
المركز القطري للامتحانات والتّقييم
מיסודן של האוניברסיטאות בישראל

Two approaches for computing standard errors and confidence intervals for correlations in the case of indirect range restriction

Research Report

RR 20-01

February 2020

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Research Report

RR-20-01

ISBN:978-965-502-215-5

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Abstract

A frequent topic of psychological research is the estimation of the correlation between two variables from a sample that underwent a selection process based on a third variable. Due to indirect range restriction, the sample correlation is a biased estimator of the population correlation, and a correction formula is used (Thorndike, 1949). In the past, bootstrap standard error and confidence intervals for the corrected correlations were examined with normal data (Li, Chan, & Cui, 2011). The present study proposes a large-sample estimate (an analytic method) for the standard error, and a corresponding confidence interval for the corrected correlation. Monte Carlo simulation studies involving both normal and nonnormal data were conducted to examine the empirical performance of the bootstrap and analytic methods. Results indicated that with both normal and nonnormal data, the bootstrap standard error and confidence interval were generally accurate across simulation conditions (restricted sample size, selection ratio, and population correlations) and outperformed estimates of the analytic method. However, with certain combinations of distribution type and model conditions, the analytic method has an advantage, offering reasonable estimates of the standard error and confidence interval without resorting to the bootstrap procedure's computer-intensive approach.

1. Introduction

A frequent topic of psychological research is the estimation of the population correlation between two variables (X and Y) on the basis of a sample that underwent a selection process (Sackett & Yang, 2000). For example, the validity of the Graduate Record Examination (GRE) for predicting performance in graduate school can be estimated only with samples of students *admitted* to the graduate program (the restricted sample). However, the goal is to estimate the validity of the GRE when used in the population of *applicants* to the graduate program. The problem is that the correlation between the GRE (X) and performance in graduate school (Y) obtained from the (available) restricted sample underestimates the correlation we would obtain from the (not available) applicant population. Hence, this biased sample correlation has to be corrected to provide a more valid population estimate.

The most common selection scenarios in personnel selection and higher education are *direct range restriction* (DRR) and *indirect range restriction* (IRR). In a DRR scenario, selection is based directly on the predictor variable X, whereas in an IRR scenario, selection is based on another variable Z. In this example, the DRR scenario refers to a case where applicants have been selected directly on the basis of the GRE. The IRR scenario refers to a case where applicants have been selected using another variable that is correlated with the GRE (such as undergraduate grade point average).

Thorndike (1949), following Pearson (1903), proposed formulas to correct the sample correlation under different scenarios of range restriction. Following this, several investigations of the accuracy of the correction formulas under a variety of circumstances (e.g., Greener & Osburn, 1979, 1980; Gross & Fleischman, 1983; Holmes, 1990) have been published.

Another issue pertaining to correlations corrected for range restriction is sampling error and its implications for confidence intervals and significance tests. Such indices are crucial for making meaningful evaluation possible. According to the *Standards for Educational and Psychological Testing*, "When effect size measures … are used to draw inferences that go beyond describing the sample … on which data have been collected, indices of the degree of uncertainty associated with these measures … should be reported … Standard errors or confidence intervals provide more information and thus are preferred in place of, or as supplements to, significance testing" (AERA, APA & NCME , 2014, p. 29). Consistent with this standpoint, the present study adds to and refines the existing body of knowledge regarding the best way to estimate standard errors (SEs) and confidence intervals (CIs) for correlations corrected for IRR.

Chan & Chan (2004) developed a bootstrap procedure for estimating SEs and CIs for correlations corrected for DRR. They compared its empirical performance with the performance of two formulas for the SEs (Fisher, 1954; Bobko & Rieck, 1980) and the respective CIs of the correlations in simulation studies involving both normal and nonnormal data.

Li et al. (2011) used a bootstrap procedure for estimating the SEs and CIs for correlations corrected for IRR. The empirical performance of the proposed procedure was examined in a simulation study with normal data.

The present study expands upon the work of Li et al. (2011) in three respects: (a) it presents an analytic method for estimating the SE and CI for the correlations corrected for IRR. The analytic method uses a formula, which gives a theory-based large-sample estimate of the SE. The empirical performance of the formula is compared with that of the bootstrap procedure; (b) it examines the performance of the formula and the bootstrap procedure with nonnormal data; and (c) it adds an additional evaluation criterion – the width of the CIs – for the performance of the different methods, which complements the information provided by the coverage probability: given constant coverage, as the width of the CI decreases, the accuracy of the estimate increases. Thus, a small width is desirable (Gross, 1976).

1.1. Correlations corrected for IRR

In the framework of IRR, the correction formula (involving the assumptions of linearity and homoscedasticity of error distribution) is as follows:

$$R_{XY} = \frac{r_{XY} + (k^2 - 1) r_{XZ} r_{YZ}}{\sqrt{1 + (k^2 - 1) r_{XZ}^2}\sqrt{1 + (k^2 - 1) r_{YZ}^2}} \qquad (1)$$

where r_{XY}, r_{XZ}, and r_{YZ} are the observed (restricted) correlations between X and Y, X and Z, and Y and Z, respectively; and $k^2 = S_Z^2 / s_Z^2$, where S_Z^2 is the variance of unrestricted Z, and s_Z^2 is the variance of restricted Z (Thorndike, 1949).

1.2. A formula for the standard error of correlations corrected for IRR

A formula of a large-sample estimate of the SE of correlations corrected for IRR was derived on the basis of the same principle used to derive the large-sample estimate of the SE of correlations corrected for DRR (Bobko & Rieck, 1980). It should be noted that a similar formula for IRR appeared in Allen & Dunbar (1990). However, one of its elements is incompatible with the following presentation.

Based on the general expression for the variance of a function of correlations, derived by Kendall & Stuart (1969), the formula of the variance of R_{XY} is

$$\mathrm{VAR}(R_{XY}) = \left(\frac{\partial R_{XY}}{\partial r_{XY}}\right)^2 \mathrm{VAR}(r_{XY}) + \left(\frac{\partial R_{XY}}{\partial r_{XZ}}\right)^2 \mathrm{VAR}(r_{XZ}) + \left(\frac{\partial R_{XY}}{\partial r_{YZ}}\right)^2 \mathrm{VAR}(r_{YZ})$$
$$+2\left(\frac{\partial R_{XY}}{\partial r_{XY}}\right)\left(\frac{\partial R_{XY}}{\partial r_{XZ}}\right)\mathrm{COV}(r_{XY}, r_{XZ}) + 2\left(\frac{\partial R_{XY}}{\partial r_{XY}}\right)\left(\frac{\partial R_{XY}}{\partial r_{YZ}}\right)\mathrm{COV}(r_{XY}, r_{YZ})$$
$$+2\left(\frac{\partial R_{XY}}{\partial r_{XZ}}\right)\left(\frac{\partial R_{XY}}{\partial r_{YZ}}\right)\mathrm{COV}(r_{XZ}, r_{YZ}) \qquad (2)$$

with the following elements substituted in Equation 2:

(a) the partial derivatives of R_{XY} with respect to each observed correlation:

$$\frac{\partial R_{XY}}{\partial r_{XY}} = \frac{1}{\sqrt{1+(k^2-1)r_{XZ}^2}\sqrt{1+(k^2-1)r_{YZ}^2}} \qquad (3)$$

$$\frac{\partial R_{XY}}{\partial r_{XZ}} = \frac{(k^2-1)}{\sqrt{1+(k^2-1)r_{YZ}^2}\left(\sqrt{1+(k^2-1)r_{XZ}^2}\right)^3}[r_{YZ} - r_{XY}r_{XZ}] \qquad (4)$$

$$\frac{\partial R_{XY}}{\partial r_{YZ}} = \frac{(k^2-1)}{\sqrt{1+(k^2-1)r_{XZ}^2}\left(\sqrt{1+(k^2-1)r_{YZ}^2}\right)^3}[r_{XZ} - r_{XY}r_{YZ}] \qquad (5)$$

(b) the asymptotic variances of the observed correlations (Fisher, 1954):

$$\mathrm{VAR}(r_{XY}) = \frac{\left(1-r_{XY}^2\right)^2}{n-1} \qquad (6)$$

$$\mathrm{VAR}(r_{XZ}) = \frac{\left(1-r_{XZ}^2\right)^2}{n-1} \qquad (7)$$

$$\mathrm{VAR}(r_{YZ}) = \frac{\left(1-r_{YZ}^2\right)^2}{n-1} \qquad (8)$$

and (c) the asymptotic covariances of the observed correlations (Dunn & Clark, 1969):

$$\text{COV}(r_{XY}, r_{XZ}) = \frac{1}{n-1} * \left[r_{YZ}(1 - r_{XY}^2 - r_{XZ}^2) - \frac{1}{2} r_{XY} r_{XZ}(1 - r_{XY}^2 - r_{XZ}^2 - r_{YZ}^2)\right] \quad (9)$$

$$\text{COV}(r_{XY}, r_{YZ}) = \frac{1}{n-1} * \left[r_{XZ}(1 - r_{XY}^2 - r_{YZ}^2) - \frac{1}{2} r_{XY} r_{YZ}(1 - r_{XY}^2 - r_{XZ}^2 - r_{YZ}^2)\right] \quad (10)$$

$$\text{COV}(r_{XZ}, r_{YZ}) = \frac{1}{n-1} * \left[r_{XY}(1 - r_{XZ}^2 - r_{YZ}^2) - \frac{1}{2} r_{XZ} r_{YZ}(1 - r_{XY}^2 - r_{XZ}^2 - r_{YZ}^2)\right] \quad (11)$$

A demonstration of how Equation 2 is collapsed to the formula of the SE of correlations corrected for DRR in the private case where Z=X is presented in Appendix A.

A 100(1-α)% formula-based standard interval (FSI) for ρ_{XY} is

$$\left[R_{XY} \pm Z_{(1-\alpha/2)} SE_F\right] \quad (12)$$

with SE_F being the square root of $VAR(R_{XY})$ computed in Equation 2.

1.3. A bootstrap procedure for estimating the standard error and confidence intervals

On the basis of studies conducted by Chan & Chan (2004) and Li et al. (2011), we used a bootstrap procedure for estimating the SE and two types of CIs. A detailed description of the bootstrap procedure can be found in Chan & Chan (2004) and Li et al. (2011).

Generally, the data observed under IRR is composed of two sub-samples: a selected (restricted) sub-sample of size n_r with scores on all 3 variables (Z_r, X_r, Y_r) and an unselected sub-sample of size n_u with scores on the selection variable (Z_u) only. A bootstrap sample is generated by randomly resampling with replacement n_r observations from the restricted sub-sample and n_u additional observations from the unselected sub-sample. Following this, the corrected correlation, R_{XY}, is calculated using Equation 1 for the bootstrap sample. This process is repeated B times.

The bootstrap estimate for the SE of the corrected correlation, SE_B, is the standard deviation of the corrected correlation across the B bootstrap samples:

$$SE_B = \sqrt{\sum_{b=1}^{B}[R^*_{XY}(b) - R^*_{XY}(.)]^2/(B-1)} \quad (13)$$

where $R^*_{XY}(b)$ is the corrected correlation in the bootstrap sample b, and $R^*_{XY}(.) = (\sum_{b=1}^{B} R^*_{XY}(b))/B$ is the mean of the B bootstrap corrected correlations.

Two CIs based on the bootstrap samples were constructed: a bootstrap standard interval (BSI) and a bootstrap percentile interval (BPI):

A 100(1-α)% BSI for ρ_{XY} is

$$\left[\mathrm{R_{XY}} \pm \mathrm{Z}_{(1-\alpha/2)}\mathrm{SE_B}\right] \tag{14}$$

To construct the BPI, the B bootstrap corrected correlations are rank-ordered, such that $\mathrm{R^*}_{XY}[1] \leq \mathrm{R^*}_{XY}[2] \leq \cdots \leq \mathrm{R^*}_{XY}[B]$ represent the ordered bootstrap corrected correlations. A 100(1-α)% bootstrap percentile interval for ρ_{XY} is

$$(\mathrm{R^*}_{XY}[l], \mathrm{R^*}_{XY}[u]) \tag{15}$$

where $l = B \cdot \alpha/2$ and $u = B(1 - \alpha/2)$. If l and u are non-integers, they are rounded to the integers $l' < l$ and $u' > u$.

2. Simulation study 1: Normal data

2.1. Method

2.1.1. Design

In order to enable replicability and comparability with Li et al. (2011), the same three factors, with the same values, were manipulated.

Restricted sample size (n_r) was manipulated at values of 20, 60, and 100.

Selection ratio (π), the ratio of the restricted to the total sample size (N) (i.e., $\pi = n_r/N$) was manipulated at .10, .30, and .50.

Population correlations (ρ_{XY}, ρ_{XZ}, and ρ_{YZ}) were considered at nine levels: ρ_{XY} was manipulated at .20, .50, and .80, while ρ_{XZ} and ρ_{YZ} were manipulated at three levels: consistent and less sizable (CL: .20, .30), inconsistent (IC: .20, .60), and consistent and more sizable (CM: .80, .60).

The three factors were factorially combined to provide 3 X 3 X 9 = 81 simulation conditions.

2.1.2. Procedure

For each condition, 1,000 random samples of size $N = n_r/\pi$ were generated from a multivariate normal distribution with population correlations ρ_{XY}, ρ_{XZ}, and ρ_{YZ}. The mean and variance of the variables were fixed at 0 and 1, respectively. A demonstration of how to construct three random variables with a predetermined distribution for one of them and a given covariance matrix is presented in Appendix B.

Within each sample, the n_r highest observations on Z constituted the restricted sub-sample, and the $N - n_r$ lowest observations on Z constituted the unselected sub-sample.

2.1.3. Data analysis

In each sample, the following statistics were computed: $\mathrm{R_{XY}}$, $\mathrm{SE_F}$, $\mathrm{SE_B}$, FSI, BSI, BPI. The number of bootstrap replications (needed for the computation of $\mathrm{SE_B}$, BSI, and BPI) was B=2,000 (Chan & Chan, 2004; Li et al., 2011).

In order to enable replicability and comparability with Li et al. (2011), the same evaluation criteria were computed (with the addition of CI width). The evaluation criteria were:

Accuracy of correlations corrected for range restriction. The average corrected correlation $\overline{\mathrm{R_{XY}}}$ was computed across the 1,000 replications. The difference between $\overline{\mathrm{R_{XY}}}$ and the population correlation ρ_{XY} was evaluated in terms of percentage bias:

$\mathrm{Bias}_R = (\overline{\mathrm{R_{XY}}} - \rho_{XY})/\rho_{XY} \times 100\%$,
where $\overline{\mathrm{R_{XY}}} = \sum_{t=1}^{1000} \mathrm{R_{XY}}(t)/1000$.
The mean absolute percentage error of the corrected correlation ($\mathrm{MAPE_R}$) was computed as the average of the absolute value of Bias_R across all model conditions.

Accuracy of different standard-error estimates. The average of each of the two SE estimates was computed across the 1,000 replications. The difference between this average and the empirical standard deviation of the corrected correlations across the 1,000 replications was evaluated in terms of percentage bias:

$\mathrm{Bias}_{SE_F} = (\overline{\mathrm{SE_F}} - SD_E)/SD_E \times 100\%$ and $\mathrm{Bias}_{SE_B} = (\overline{\mathrm{SE_B}} - SD_E)/SD_E \times 100\%$,
where $\overline{\mathrm{SE_F}} = \sum_{t=1}^{1000} \mathrm{SE_F}(t)/1000$, $\overline{\mathrm{SE_B}} = \sum_{t=1}^{1000} \mathrm{SE_B}(t)/1000$
and $SD_E = \sqrt{\sum_{t=1}^{1000} [\mathrm{R_{XY}}(t) - \overline{\mathrm{R_{XY}}}]^2/1000}$.
The mean absolute percentage error of the SE (MAPE_{SE_F} or MAPE_{SE_B}) was computed as the average of the absolute value of Bias_{SE_F} or Bias_{SE_B}, respectively, across all model conditions.

Coverage probability of confidence intervals. For each of the three CIs, the percentage of times that the CI contained ρ_{XY} among the 1,000 replications was calculated.

The average of the coverage probability across all model conditions was computed.

Confidence interval width. For each of the three CIs, the CI width was calculated by subtracting the lower limit from the upper limit and averaging it across the 1,000 replications.

The average width across all model conditions was computed.

Assessment of the evaluation criteria. We determined the acceptability of the different evaluation criteria as follows (Chan & Chan, 2004; Li et al., 2011): For Bias_R, Bias_{SE_B}, and Bias_{SE_F}, the parameter estimate was considered excellent if bias was within $\pm5\%$ and reasonable if bias was within $\pm10\%$. For the coverage probability of the CIs, a coverage probability falling within [.92, .97], which allows for sampling error, may be considered acceptable (Li et al., 2011). Based on conventional thinking (Rothman, Greenland & Lash, 2008), which maintains that overcoverage is preferable to undercoverage, we treated a conservative interval as acceptable, too.

2.2. Results

2.2.1. Accuracy of correlations corrected for IRR

The corrected correlations were generally accurate. Overall, the MAPE_R was 4.85%, showing an excellent fit. Of the 81 conditions, 71 produced a Bias_R within $\pm10\%$. The direction of the bias was generally negative (Table 1).

Since no interaction effect was found, the effect of each factor can be discussed separately. The accuracy of the corrected correlation, R_XY, improved as n_r, π, and ρ_{XY} increased, as was found by Chan & Chan (2004) for correlations corrected for DRR and by Li et al. (2011) for correlations corrected for IRR. As for ρ_{XZ} and ρ_{YZ}, the most accurate estimation occurred under CL, then IC, and finally CM, as was found by Li et al. (2011).

Table 1 - Mean of correlations corrected for IRR ($\overline{R_{XY}}$) and percentage bias ($Bias_R$) in 81 simulation conditions for normal data

		Consistent and less sizable (CL): ρ_{XZ}=.20, ρ_{YZ}=.30					
		ρ_{XY} = .20		ρ_{XY} = .50		ρ_{XY} = .80	
n_r	π	$\overline{R_{XY}}$	$Bias_R$	$\overline{R_{XY}}$	$Bias_R$	$\overline{R_{XY}}$	$Bias_R$
20	0.1	.175	**-12.3**	.471	-5.9	.497	-0.6
	0.3	.188	-5.8	.483	-3.4	.778	-2.7
	0.5	.186	-7.1	.484	-3.1	.788	-1.5
60	0.1	.187	-6.3	.484	-3.2	.792	-1.0
	0.3	.198	-1.1	.494	-1.1	.792	-1.1
	0.5	.198	-1.1	.496	-0.8	.796	-0.5
100	0.1	.192	-4.1	.491	-1.7	.797	-0.4
	0.3	.199	-0.7	.497	-0.6	.796	-0.6
	0.5	.198	-1.0	.497	-0.6	.798	-0.3
		Inconsistent (IC): ρ_{XZ}=.20, ρ_{YZ}=.60					
		ρ_{XY} = .20		ρ_{XY} = .50		ρ_{XY} = .80	
n_r	π	$\overline{R_{XY}}$	$Bias_R$	$\overline{R_{XY}}$	$Bias_R$	$\overline{R_{XY}}$	$Bias_R$
20	0.1	.155	**-22.3**	.466	-6.7	.799	-0.2
	0.3	.175	**-12.3**	.478	-4.4	.791	-1.1
	0.5	.192	-4.2	.492	-1.6	.798	-0.3
60	0.1	.175	**-12.7**	.480	-4.0	.796	-0.6
	0.3	.189	-5.5	.490	-2.0	.797	-0.4
	0.5	.198	-1.0	.495	-1.1	.796	-0.5
100	0.1	.187	-6.7	.489	-2.1	.796	-0.5
	0.3	.194	-2.8	.495	-1.0	.798	-0.2
	0.5	.198	-1.2	.496	-0.9	.796	-0.5
		Consistent and more sizable (CM): ρ_{XZ}=.80, ρ_{YZ}=.60					
		ρ_{XY} = .20		ρ_{XY} = .50		ρ_{XY} = .80	
n_r	π	$\overline{R_{XY}}$	$Bias_R$	$\overline{R_{XY}}$	$Bias_R$	$\overline{R_{XY}}$	$Bias_R$
20	0.1	.092	**-54.1**	.400	**-20.0**	.747	-6.6
	0.3	.132	**-33.9**	.454	-9.1	.766	-4.2
	0.5	.163	**-18.4**	.478	-4.5	.783	-2.1
60	0.1	.171	**-14.3**	.468	-6.3	.779	-2.6
	0.3	.183	-8.5	.487	-2.6	.787	-1.6
	0.5	.189	-5.3	.492	-1.6	.793	-0.9
100	0.1	.175	**-12.3**	.476	-4.8	.785	-1.8
	0.3	.191	-4.5	.491	-1.7	.793	-0.9
	0.5	.193	-3.7	.495	-1.1	.796	-0.5

Note. ρ_{XY}, ρ_{XZ}, and ρ_{YZ} are the population correlations between variables X and Y, X and Z, and Y and Z, respectively. n_r is the restricted sample size. π is the selection ratio. $\overline{R_{XY}}$ is the mean of correlations corrected for IRR. $Bias_R$ is the percentage bias of $\overline{R_{XY}}$. $Bias_R$s that are not within ±10% are shown in bold.

2.2.2. Accuracy of different standard-error estimates

Overall, $\mathrm{MAPE}_{SE_B}$was 3.82% and MAPE_{SE_F} was 5.74%, showing an excellent and a reasonable estimation by the bootstrap procedure and the formula, respectively. Of the 81 conditions, 75 and 69 produced Bias_{SE_B} and Bias_{SE_F}, respectively, within $\pm 10\%$ (Table 2).

The effect of the different factors we manipulated was as follows: the performance of both the bootstrap procedure and the formula improved (smaller biases) as n_r increased, as was found by Chan & Chan (2004) and Li et al. (2011), and as π increased. As for ρ_{XY}, lower values (0.2) were, on average across conditions, associated with a better performance and higher values (0.8) with worse performance. As for the two other population correlations, the worse performance was associated with CL, as was indicated by Li et al. (2011). In general, the directions of the effects described above were similar in Bias_{SE_B} and Bias_{SE_F}. However, the magnitudes of these effects with respect to the accuracy of the two estimates differed. In particular, the effect of n_r on Bias_{SE_F} was much more pronounced (especially when switching from n_r =20 to n_r =60 or to n_r =100) than its effect on Bias_{SE_B}. Thus, as noted by Chan & Chan (2004), the superiority of the bootstrap procedure over the formula gradually vanished as n_r increased.

Table 2 - Mean standard errors ($\overline{SE_F}, \overline{SE_B}$) and percentage biases ($Bias_{SE_F}, Bias_{SE_B}$) of two standard-error estimates for correlations corrected for IRR in 81 simulation conditions for normal data

		Consistent and less sizable (CL): ρ_{XZ}=.20, ρ_{YZ}=.30											
		ρ_{XY} = .20				ρ_{XY} = .50				ρ_{XY} = .80			
n_r	π	$\overline{SE_F}$	$\overline{SE_B}$	$Bias_{SE_F}$	$Bias_{SE_B}$	$\overline{SE_F}$	$\overline{SE_B}$	$Bias_{SE_F}$	$Bias_{SE_B}$	$\overline{SE_F}$	$\overline{SE_B}$	$Bias_{SE_F}$	$Bias_{SE_B}$
20	0.1	.356	.316	**25.1**	**11.1**	.291	.265	**23.3**	**12.1**	.164	.152	**24.8**	**15.1**
	0.3	.284	.268	**11.0**	4.5	.234	.223	**13.0**	7.4	.126	.124	**14.8**	**12.2**
	0.5	.256	.246	4.2	0.1	.209	.203	9.3	5.9	.110	.110	**16.1**	**16.2**
60	0.1	.172	.167	6.8	3.7	.138	.135	9.3	7.3	.070	.069	**13.5**	**12.5**
	0.3	.149	.147	5.1	3.4	.118	.116	5.9	4.8	.059	.058	6.6	6.0
	0.5	.139	.137	0.6	-0.8	.109	.108	1.9	1.0	.054	.054	2.9	2.5
100	0.1	.127	.126	1.6	0.4	.100	.099	4.5	4.1	.049	.049	8.4	8.5
	0.3	.113	.111	3.9	2.5	.088	.087	5.5	4.7	.043	.043	6.2	5.8
	0.5	.106	.105	-0.5	-1.4	.083	.082	0.8	0.1	.041	.040	2.6	2.2
		Inconsistent (IC): ρ_{XZ}=.20, ρ_{YZ}=.60											
		ρ_{XY} = .20				ρ_{XY} = .50				ρ_{XY} = .80			
n_r	π	$\overline{SE_F}$	$\overline{SE_B}$	$Bias_{SE_F}$	$Bias_{SE_B}$	$\overline{SE_F}$	$\overline{SE_B}$	$Bias_{SE_F}$	$Bias_{SE_B}$	$\overline{SE_F}$	$\overline{SE_B}$	$Bias_{SE_F}$	$Bias_{SE_B}$
20	0.1	.354	.320	9.0	-1.6	.304	.272	**14.7**	2.7	.175	.149	**26.9**	7.5
	0.3	.300	.280	3.9	-3.0	.253	.234	8.3	0.2	.140	.127	**10.1**	-0.0
	0.5	.268	.255	0.1	-4.8	.222	.212	7.0	2.1	.122	.115	**10.7**	4.5
60	0.1	.200	.188	-0.3	-6.3	.165	.154	0.4	-6.1	.090	.083	3.8	-5.1
	0.3	.170	.164	-2.5	-5.7	.138	.133	-0.7	-4.3	.075	.071	2.9	-2.3
	0.5	.155	.151	0.2	-3.0	.126	.122	0.9	-2.4	.067	.064	2.5	-1.8
100	0.1	.155	.148	-1.3	-5.6	.126	.120	-2.4	-7.1	.069	.064	-0.4	-6.8
	0.3	.132	.129	-1.8	-4.2	.107	.104	0.3	-2.1	.057	.055	0.0	-3.8
	0.5	.120	.118	0.1	-1.6	.097	.095	0.8	-0.9	.051	.050	1.7	-1.1
		Consistent and more sizable (CM): ρ_{XZ}=.80, ρ_{YZ}=.60											
		ρ_{XY} = .20				ρ_{XY} = .50				ρ_{XY} = .80			
n_r	π	$\overline{SE_F}$	$\overline{SE_B}$	$Bias_{SE_F}$	$Bias_{SE_B}$	$\overline{SE_F}$	$\overline{SE_B}$	$Bias_{SE_F}$	$Bias_{SE_B}$	$\overline{SE_F}$	$\overline{SE_B}$	$Bias_{SE_F}$	$Bias_{SE_B}$
20	0.1	.368	.336	7.9	-1.5	.334	.307	3.5	-4.7	.209	.191	10.0	0.6
	0.3	.304	.288	3.2	-2.3	.264	.255	3.0	-0.5	.160	.157	5.1	3.3
	0.5	.268	.254	0.8	-4.2	.227	.223	3.7	1.8	.133	.131	9.5	8.2
60	0.1	.193	.183	5.9	0.2	.173	.167	4.5	1.2	.101	.098	6.3	2.9
	0.3	.163	.155	3.8	-1.5	.141	.137	3.5	0.5	.079	.077	5.6	3.2
	0.5	.149	.142	7.2	1.7	.125	.122	4.4	1.4	.068	.067	2.4	0.9
100	0.1	.147	.138	9.5	3.1	.130	.126	0.6	-2.4	.074	.072	0.5	-2.0
	0.3	.125	.117	4.4	-1.8	.107	.103	2.9	-0.6	.058	.057	4.1	2.3
	0.5	.114	.109	3.9	-1.3	.096	.093	4.2	1.4	.051	.050	3.0	1.4

Note. ρ_{XY}, ρ_{XZ}, and ρ_{YZ} are the population correlations between variables X and Y, X and Z, and Y and Z, respectively. n_r is the restricted sample size. π is the selection ratio. $\overline{SE_F}$ and $\overline{SE_B}$ are the means of the formula and the bootstrap SEs, respectively, across the 1,000 replications. $Bias_{SE_F}$ and $Bias_{SE_B}$ are the percentage of bias of $\overline{SE_F}$ and $\overline{SE_B}$, respectively. $Bias_{SE_F}$s and $Bias_{SE_B}$s that are not within ±10% are shown in bold.

2.2.3. Accuracy of different confidence intervals

Of the three types of CIs, BPI exhibited the best performance. It produced coverage probability above the lower limit of the nominal range in all 81 model conditions, with a mean of .95 (Table 3). BSI showed slightly poorer results. In 63 of the 81 conditions, the coverage probability was within the nominal range, with a mean of .93. FSI showed still poorer performance. In 58 of the 81 conditions, the coverage probability was within the nominal range, with a mean of .93.

Table 3 - Coverage probabilities of three confidence intervals for correlations corrected for IRR in 81 simulation conditions for normal data

		Consistent and less sizable (CL): ρ_{XZ}=.20, ρ_{YZ}=.30								
		$\rho_{XY}=.20$			$\rho_{XY}=.50$			$\rho_{XY}=.80$		
n_r	π	FSI	BSI	BPI	FSI	BSI	BPI	FSI	BSI	BPI
20	0.1	0.92	0.94	0.97	0.93	0.95	0.98	0.92	0.96	0.98
	0.3	0.93	0.93	0.97	0.92	0.94	0.96	**0.91**	0.94	0.96
	0.5	0.92	0.92	0.95	0.93	0.93	0.95	**0.91**	0.93	0.95
60	0.1	0.95	0.95	0.97	0.96	0.96	0.96	0.95	0.96	0.97
	0.3	0.95	0.95	0.97	0.95	0.95	0.96	0.95	0.94	0.96
	0.5	0.94	0.93	0.94	0.94	0.94	0.94	0.94	0.94	0.95
100	0.1	0.95	0.94	0.96	0.95	0.95	0.96	0.95	0.96	0.98
	0.3	0.95	0.95	0.96	0.96	0.96	0.96	0.95	0.95	0.97
	0.5	0.94	0.93	0.94	0.95	0.94	0.96	0.95	0.95	0.96
		Inconsistent (IC): ρ_{XZ}=.20, ρ_{YZ}=.60								
		$\rho_{XY}=.20$			$\rho_{XY}=.50$			$\rho_{XY}=.80$		
n_r	π	FSI	BSI	BPI	FSI	BSI	BPI	FSI	BSI	BPI
20	0.1	**0.89**	**0.91**	0.97	**0.89**	0.92	0.97	**0.82**	**0.91**	0.95
	0.3	**0.89**	**0.90**	0.95	**0.90**	0.92	0.96	**0.86**	**0.89**	0.93
	0.5	**0.90**	**0.91**	0.94	**0.91**	0.92	0.95	**0.88**	**0.90**	0.94
60	0.1	0.92	0.93	0.95	0.92	0.92	0.95	**0.89**	**0.89**	0.95
	0.3	0.92	0.92	0.94	0.93	0.92	0.95	**0.91**	**0.91**	0.94
	0.5	0.93	0.92	0.94	0.93	0.92	0.94	0.93	0.93	0.94
100	0.1	0.93	**0.91**	0.94	0.92	**0.91**	0.94	**0.90**	**0.90**	0.93
	0.3	0.95	0.94	0.95	0.95	0.94	0.95	0.92	0.92	0.94
	0.5	0.95	0.93	0.94	0.95	0.94	0.95	0.94	0.94	0.94
		Consistent and more sizable (CM): ρ_{XZ}=.80, ρ_{YZ}=.60								
		$\rho_{XY}=.20$			$\rho_{XY}=.50$			$\rho_{XY}=.80$		
n_r	π	FSI	BSI	BPI	FSI	BSI	BPI	FSI	BSI	BPI
20	0.1	**0.89**	**0.90**	0.94	**0.88**	**0.90**	0.95	**0.87**	0.93	0.94
	0.3	**0.91**	**0.90**	0.93	**0.89**	**0.91**	0.93	**0.88**	0.93	0.94
	0.5	**0.91**	**0.89**	0.92	**0.90**	**0.91**	0.94	**0.88**	**0.91**	0.93
60	0.1	0.94	0.93	0.94	0.94	0.94	0.94	0.92	0.93	0.93
	0.3	0.95	0.94	0.95	0.94	0.94	0.94	0.94	0.94	0.94
	0.5	0.96	0.94	0.94	0.94	0.94	0.95	0.93	0.93	0.94
100	0.1	0.96	0.95	0.95	0.94	0.94	0.95	0.93	0.93	0.94
	0.3	0.95	0.93	0.94	0.94	0.93	0.94	0.94	0.94	0.94
	0.5	0.95	0.94	0.94	0.95	0.94	0.95	0.94	0.95	0.94

Note. ρ_{XY}, ρ_{XZ}, and ρ_{YZ} are the population correlations between variables X and Y, X and Z, and Y and Z, respectively. n_r is the restricted sample size. π is the selection ratio. FSI is the 95% formula standard interval. BSI is the 95% bootstrap standard interval. BPI is the 95% bootstrap percentile interval. Coverage probabilities that are below .92 are shown in bold.

The effect of the different factors on the coverage probabilities of the three CIs was not identical. As mentioned above, BPI performed well under all conditions, while the performance of the other CIs, especially that of FSI, varied across the levels of the different factors. Thus, for example, FSI performed poorly at n_r =20 (where in only 7 of the 27 cases was the coverage probability within the nominal range) and much better at n_r =60 or n_r =100 (where in 25 and 26, respectively, of the 27 cases was the coverage probability within the nominal range). The combination of ρ_{XZ} and ρ_{YZ} also had an effect on the coverage probability of FSI (better coverage probability at CL) as did, though to a lesser extent, ρ_{XY} (better coverage probability at $\rho_{XY} = 0.2$ or $\rho_{XY} = 0.5$).

2.2.4. Confidence interval width

Overall, the average width of the CIs was sizable (Table 4), with minor differences among the three types of CIs: it ranged from 0.560 (for BPI) to 0.572 (for FSI). For comparison, the width of a 95% CI of an observed (unrestricted) correlation with $r_{XY} = .50$ and n_r = 60 is 0.389.

If we limit ourselves to the comparison between symmetric (e.g., FSI) and non-symmetric (BPI) CIs only (since the two symmetric CIs (should) have, by definition, equal width under identical coverage probability), and to CIs with a coverage probability above the lower limit of the nominal range (58 and 81 conditions for FSI and BPI, respectively), we find that the average width of FSI and BPI was 0.560 and 0.493, respectively. And since the distribution of the values of the factors we manipulated was not identical in the computation of the average width reported above, we further limited ourselves to the conditions where the coverage probabilities of both CIs were within the nominal range (58). The average width of the CIs under these conditions was 0.493 and 0.483 for FSI and BPI, respectively. Thus, in cases where FSI had an acceptable coverage probability, minor differences were found among the CIs, with BPI somewhat superior in terms of the width of the CIs.

Table 4 - Mean width of three confidence intervals for correlations corrected for IRR in 81 simulation conditions for normal data

		Consistent and less sizable (CL): ρ_{XZ}=.20, ρ_{YZ}=.30								
		$\rho_{XY}=.20$			$\rho_{XY}=.50$			$\rho_{XY}=.80$		
n_r	π	FSI	BSI	BPI	FSI	BSI	BPI	FSI	BSI	BPI
20	0.1	1.323	1.240	1.195	1.060	1.038	1.009	0.555	0.594	0.575
	0.3	1.082	1.049	1.028	0.871	0.873	0.858	0.444	0.485	0.473
	0.5	0.985	0.963	0.950	0.789	0.795	0.785	0.396	0.432	0.423
60	0.1	0.664	0.657	0.655	0.524	0.530	0.528	0.260	0.272	0.270
	0.3	0.579	0.574	0.572	0.454	0.456	0.454	0.222	0.228	0.227
	0.5	0.541	0.536	0.534	0.423	0.424	0.422	0.207	0.211	0.210
100	0.1	0.493	0.492	0.492	0.384	0.390	0.390	0.188	0.194	0.193
	0.3	0.440	0.437	0.435	0.342	0.343	0.342	0.167	0.169	0.169
	0.5	0.414	0.411	0.410	0.323	0.323	0.323	0.157	0.159	0.158
		Inconsistent (IC): ρ_{XZ}=.20, ρ_{YZ}=.60								
		$\rho_{XY}=.20$			$\rho_{XY}=.50$			$\rho_{XY}=.80$		
n_r	π	FSI	BSI	BPI	FSI	BSI	BPI	FSI	BSI	BPI
20	0.1	1.324	1.253	1.204	1.097	1.068	1.030	0.575	0.582	0.553
	0.3	1.141	1.098	1.068	0.932	0.916	0.893	0.487	0.499	0.482
	0.5	1.030	1.000	0.981	0.833	0.831	0.815	0.431	0.451	0.439
60	0.1	0.770	0.738	0.730	0.623	0.604	0.597	0.332	0.324	0.317
	0.3	0.659	0.644	0.638	0.530	0.522	0.517	0.280	0.278	0.274
	0.5	0.605	0.590	0.586	0.484	0.476	0.473	0.253	0.252	0.250
100	0.1	0.599	0.580	0.576	0.481	0.470	0.467	0.260	0.253	0.250
	0.3	0.515	0.505	0.503	0.412	0.408	0.405	0.218	0.215	0.213
	0.5	0.470	0.464	0.462	0.375	0.373	0.371	0.196	0.195	0.194
		Consistent and more sizable (CM): ρ_{XZ}=.80, ρ_{YZ}=.60								
		$\rho_{XY}=.20$			$\rho_{XY}=.50$			$\rho_{XY}=.80$		
n_r	π	FSI	BSI	BPI	FSI	BSI	BPI	FSI	BSI	BPI
20	0.1	1.360	1.315	1.256	1.186	1.205	1.154	0.673	0.751	0.719
	0.3	1.149	1.128	1.105	0.964	1.001	0.975	0.539	0.615	0.592
	0.5	1.022	0.997	0.985	0.846	0.874	0.859	0.458	0.514	0.499
60	0.1	0.739	0.717	0.713	0.647	0.655	0.648	0.359	0.382	0.375
	0.3	0.632	0.608	0.607	0.535	0.535	0.531	0.291	0.304	0.299
	0.5	0.580	0.555	0.554	0.482	0.477	0.475	0.253	0.261	0.259
100	0.1	0.566	0.542	0.542	0.495	0.495	0.492	0.270	0.282	0.278
	0.3	0.486	0.461	0.460	0.412	0.405	0.404	0.219	0.224	0.223
	0.5	0.446	0.426	0.425	0.371	0.365	0.364	0.192	0.195	0.194

Note. ρ_{XY}, ρ_{XZ}, and ρ_{YZ} are the population correlations between variables X and Y, X and Z, and Y and Z, respectively. n_r is the restricted sample size. π is the selection ratio. FSI is the 95% formula standard interval. BSI is the 95% bootstrap standard interval. BPI is the 95% bootstrap percentile interval.

3. Simulation study 2: Nonnormal data

3.1. Method

In this simulation study, we generated Z (the variable on which selection was based) from three types of nonnormal distribution with different degrees of skewness. In order to enable comparability with Chan & Chan's work (2004), the same three distributions were examined. The first distribution was the log-normal distribution L_n(-0.77,1), which has unit variance and positive skewness $\gamma_1 = 6.18$. The second distribution was obtained by multiplying the data in the first distribution by -1, so that Z had unit variance and negative skewness $\gamma_1 = -6.18$. The third distribution was the uniform distribution $U(0, \sqrt{12})$, which has unit variance and is symmetric with $\gamma_1 = 0$. A demonstration of how to construct three random variables with a predetermined distribution for one of them and a given covariance matrix is presented in Appendix B and is a straightforward generalization of the procedure described in Chan & Chan (2004). The rest of the study followed the same procedure as that of the first study.

3.2. Results

3.2.1. Accuracy of correlations corrected for IRR

Table 5 gives summary results regarding the accuracy of all the statistics – the corrected correlations, the 2 SEs and the 3 CIs – for nonnormal data. To complete the picture, parallel results for normal data are presented as well. The accuracy of the corrected correlations depended on the type of distribution of Z: $\mathrm{MAPE_R}$ was 3.74%, 54.05%, and 17.97% when Z was positively skewed (log-normal), negatively skewed (-1*log-normal), and uniformly distributed, respectively. In most cases, $\mathrm{R_{XY}}$ underestimated ρ_{XY} (see detailed results for nonnormal data in Tables 6, 10 and 14 in Appendix C). Thus, $\mathrm{R_{XY}}$ provided an excellent estimate of ρ_{XY} when Z was positively skewed and a serious underestimation when Z was negatively skewed. When Z was uniformly distributed, the estimate was poor, but less so than in the case of the negative skew. Chan & Chan (2004) reported similar results.

Table 5 - Summative evaluation criteria for the accuracy of the corrected correlations, the SEs and the CIs, across 81 simulation conditions, for normal and nonnormal data

Summative evaluation criteria	Statistic	Distribution of Z normal	*log-normal*	*(-1)* log-normal*	*uniform*
MAPE	$\overline{\mathrm{R}_{\mathrm{XY}}}$	4.85%	3.74%	54.05%	17.97%
	$\overline{\mathrm{SE}_{\mathrm{F}}}$	5.74%	11.07%	100.88%	27.74%
	$\overline{\mathrm{SE}_{\mathrm{B}}}$	3.82%	6.41%	4.70%	5.30%
No. of conditions with an acceptable coverage probability	FSI	58	32	1	39
	BSI	63	47	63	65
	BPI	81	64	78	81

Note. Z is the variable on which selection was based. MAPE is the mean absolute percentage error. $\overline{\mathrm{R}_{\mathrm{XY}}}$ is the correlation between X and Y corrected for IRR. $\overline{\mathrm{SE}_{\mathrm{F}}}$ and $\overline{\mathrm{SE}_{\mathrm{B}}}$ are estimators of the standard error of $\overline{\mathrm{R}_{\mathrm{XY}}}$, computed by the formula and by the bootstrap procedure, respectively. FSI, BSI, and BPI are confidence intervals for ρ_{XY} – a formula-based standard interval, a bootstrap-based standard interval and bootstrap percentile interval, respectively.

Considering the effect of the different factors we manipulated on the accuracy of R_{XY}, we found a prominent effect of ρ_{XY}, ρ_{XZ}, and ρ_{YZ}, with a similar direction for the three types of distributions (worse performance when $\rho_{XY} = 0.2$ and under CM condition). The effect of n_r and π was generally weaker, with n_r having a similar effect on the three types of distributions (as n_r increases, the accuracy of R_{XY} increases), and π having a varying effect (when Z was positively skewed, R_{XY} became less accurate as π increased, and when Z was negatively skewed or uniformly distributed, R_{XY} became more accurate as π increased). Chan & Chan (2004) reported similar results.

3.2.2. *Accuracy of different standard-error estimates*

The performance of $\overline{\mathrm{SE}_{\mathrm{F}}}$ and $\overline{\mathrm{SE}_{\mathrm{B}}}$ as estimators of SEs also depended on the type of distribution of Z (see summary results in Table 5; detailed results for nonnormal data are presented in Tables 7, 11 and 15 in Appendix C).

When Z was positively skewed, $\overline{\mathrm{SE}_{\mathrm{B}}}$ was more accurate than $\overline{\mathrm{SE}_{\mathrm{F}}}$: Overall, $\mathrm{MAPE}_{SE_F}$was 11.07% and MAPE_{SE_B} was 6.41%. The superiority of $\overline{\mathrm{SE}_{\mathrm{B}}}$ was evident mainly under the CM condition, while under the CL and IC conditions, the estimators' performance was more similar.

When Z was negatively skewed, $\overline{\mathrm{SE}_{\mathrm{B}}}$ performed excellently ($\mathrm{MAPE}_{SE_B} = 4.70\%$) while $\overline{\mathrm{SE}_{\mathrm{F}}}$ performed poorly ($\mathrm{MAPE}_{SE_F} = 100.88\%$). The superiority of $\overline{\mathrm{SE}_{\mathrm{B}}}$ was maintained under all 81 conditions.

When Z was uniformly distributed, $\overline{\mathrm{SE_B}}$ performed reasonably well ($\mathrm{MAPE}_{SE_B} = 5.30\%$) while $\overline{\mathrm{SE_F}}$ performed poorly ($\mathrm{MAPE}_{SE_F} = 27.74\%$), although better than in cases where Z was negatively skewed. The performance of $\overline{\mathrm{SE_F}}$ depended heavily on π, and to a lesser extent on n_r. Thus, with a higher selection ratio (i.e. $\pi = 0.5$) and larger sample size (i.e. n_r =60 or n_r =100), the performance of the formula was similar to that of the bootstrap procedure.

3.2.3. Accuracy of different confidence intervals

The performance of BPI and BSI when Z was negatively skewed or uniformly distributed was similar to the one observed with normal data, bearing in mind that we treat overcoverage as acceptable (see Table 5). A certain deterioration in performance was observed when Z was positively skewed. As for FSI, there were pronounced fluctuations in its performance across different types of distributions: With all the nonnormal data, its performance was worse than with normal data, and it was particularly problematic when Z was negatively skewed.

Under all types of nonnormal distributions – as with normal data – BPI performed better than BSI, which performed better than FSI. However, with positive skewness and uniform distributions, there were combinations of conditions where FSI performed rather well (see detailed results for nonnormal data in Tables 8, 12 and 16 in Appendix C). Thus, when Z was positively skewed, FSI performed satisfactorily under CL and higher n_r (60 or 100). When Z was uniformly distributed, FSI performed satisfactorily with higher π (0.3 or 0.5) and higher n_r (60 or 100).

3.2.4. Confidence interval width

As noted vis-à-vis normal data, a comparison among the CIs with respect to their width (Tables 9, 13, and 17 in Appendix C) is meaningful only when limited to conditions where the coverage probabilities of all the CIs are not below the lower limit of the nominal range. For the case where Z was positively skewed, the average width of the CIs under these 32 (of 81) model conditions was 0.314 and 0.317 for FSI and BPI, respectively. For the case where Z was negatively skewed, the coverage probabilities of FSI were practically always below the nominal range, so the average width of the CIs was not computed. For the case where Z was uniformly distributed, the average width of the CIs under these 39 (of 81) model conditions was 0.575 and 0.556 for FSI and BPI, respectively. Thus, in cases where FSI had an acceptable coverage probability, when Z was positively skewed, the two CIs were similar, and when Z was uniformly distributed, minor differences were found, with a certain superiority of BPI in terms of the width of the CIs.

4. Discussion and conclusions

Chan & Chan (2004) compared the empirical performance of a bootstrap procedure (Mendoza, Hart & Powell, 1991) with the performance of two formulas (Fisher, 1954; Bobko & Rieck, 1980) for estimating the SEs and CIs of correlations corrected for DRR, using both normal and nonnormal data. Li et al. (2011) examined the performance of the bootstrap procedure under the case of IRR, using normal data only. The present study expanded the work of Li et al. (2011) in the spirit of Chan & Chan's work (2004): We compared the performance of a bootstrap procedure with the performance of a formula based on a large-sample estimate of the SE, which we developed based on Bobko & Rieck (1980), for the case of IRR. The comparison was based on both normal and nonnormal data.

With respect to the accuracy of the correction, results showed that the corrected correlations were generally accurate when the selection variable, Z, was normally distributed or positively skewed, but they greatly underestimated ρ_{XY} when Z was negatively skewed and, to a lesser extent, when it was uniformly distributed, as was found by Chan & Chan (2004) in the case of DRR. This issue of the accuracy of the correction for range restriction formula under different conditions was addressed extensively in the literature (e.g., Brewer & Hills, 1969; Greener & Osburn, 1979, 1980; Gross & Fleischman, 1983; Held & Foley, 1994; Holmes, 1990). It should be noted that no normality assumption is required for range restriction correction formulas (Lawley, 1943; Sackett & Yang, 2000). The critical assumptions made by these formulas are linearity and homoscedasticity. The fact that the correction was unsatisfactory with certain data may be the result of the way we constructed our simulated data (see Appendix B), which led to nonlinearities and violations of homoscedasticity that were not offset (Gross, 1982).

Turning to the main topics of this article – SEs and CIs for corrected correlations – our study showed that $\mathrm{SE_B}$ was accurate with all types of data, while performance of $\mathrm{SE_F}$ was satisfactory only with normal data. In particular, it was grossly inaccurate when Z was negatively skewed. This is not surprising, given that under negative skewness, the corrected correlation itself was grossly biased, and since the computation of $\mathrm{SE_F}$ is based on the value of the corrected correlation (unlike the computation of $\mathrm{SE_B}$), it is biased, too. A direct comparison between the two estimates of the SE showed that $\mathrm{SE_B}$ was generally more accurate than $\mathrm{SE_F}$. However, with all types of distributions, except for the case where Z was negatively skewed, this superiority vanished under certain conditions: when Z was normally distributed and the sample size was large, when Z was positively skewed under CL and IC, and when Z was uniformly distributed, selection ratio was high and sample size was large.

As for CIs, BPI outperformed BSI, which outperformed FSI with all types of distributions. The coverage probabilities of BPI were generally above the lower limit of the nominal range with normal data and when Z was negatively skewed or uniformly distributed. When Z was positively skewed, the performance of BPI was not as good, but still acceptable. FSI, on the other hand, was generally inaccurate, with coverage probabilities consistently below the expected value. The performance of FSI ranged from marginally acceptable with normal data to totally inadmissible when Z was negatively skewed. As with SEs, with all types of distributions, save when Z was negatively skewed, there were certain model conditions where FSI performed as good as BPI.

We also examined CI width. The differences between CIs in this respect were generally small, although it should be noted that our comparisons were limited to conditions where the coverage probabilities of the two CIs we compared were above the lower limit of the nominal range. Aside from differences among the CIs, it is important to note that when analyzing correlations corrected for IRR, the widths of the CIs are sometimes (e.g., when Z is normally or uniformly distributed) elevated compared with the width of the CI of an observed (unrestricted) correlation. Given that in many real-life situations, researchers have small samples, awareness of the uncertainty involved in analyzing corrected correlations is important.

To conclude, we recommend that in real-life situations where, as often happens, the researcher lacks information such as type of distribution and other relevant factors, the bootstrap procedure is the better alternative for analyzing correlational data that are subject to IRR. However, we propose that since the bootstrap approach involves a resource-intensive, computer-based resampling, it would be preferable to use the approximation offered by the analytic approach, when conditions – chiefly sample size – are appropriate. With respect to other factors, the investigator often has access to at least some information necessary to make the best decision. He usually knows the selection ratio operating for his data. In addition, he might not know the type of distribution or degree of correlation among the variables in the unrestricted group, but he may be able to conjecture about them. On the basis of this information, the investigator can decide whether to apply procedures that consume fewer resources. Thus, in addition to establishing the general advantages of the bootstrap procedure for estimating the SE and CI for corrected correlations (Chan & Chan, 2004; Li et al. 2011), this study points to contexts where an analytical method offers reasonable estimates without resorting to the bootstrap procedure's computer-intensive approach. Although in most practical applications the time savings are negligible, efficiency has a theoretical advantage. In addition, many studies involve the computation of predictive validity data across many units. In such circumstances, the accumulation of time saved may matter. The analytic method was recently applied in such circumstances to an examination of the validity of medical school admissions processes (Kennet-Cohen, Turvall, Saar, & Oren, 2016).

Finally, although we have responded to some of the challenges posed in the literature (Chan & Chan, 2004; Li et al., 2011), namely, examining SEs and CIs for correlations corrected for IRR with nonnormal data, the work is not complete. First of all, it is necessary to examine other simulation conditions. We may expect, for example, that with certain conditions, such as larger sample size or $\rho_{XY} = 0$, the analytical method will have a better chance of demonstrating its advantages (e.g., Bishara & Hittner, 2012). Secondly, as Chan & Chan (2004) proposed, it is important to explore additional data distributions. One possibility would be to examine the exponential and Poisson distributions, commonly used in the applied sciences. Finally, examining the improvements gained by transformations and bias corrections is another direction for future research.

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Appendices

Appendix A:
Standard error of correlations corrected for IRR in the private case where Z=X

For the private case where Z=X:

$$\mathrm{VAR}(r_{XZ}) = \frac{(1 - r_{XZ}^2)^2}{n-1} = 0 \tag{16}$$

$$\mathrm{VAR}(r_{YZ}) = \mathrm{VAR}(r_{XY}) \tag{17}$$

$$\mathrm{COV}(r_{XY}, r_{XZ}) = \frac{1}{n-1}\left[r_{YX}(1 - r_{XY}^2 - 1^2) - \frac{1}{2} r_{XY} 1 (1 - r_{XY}^2 - 1^2 - r_{YX}^2)\right]$$

$$= \frac{1}{n-1}\left[r_{XY}(-r_{XY}^2) - \frac{1}{2} r_{XY} 1 (-r_{XY}{}^2 - r_{XY}{}^2)\right]$$

$$= \frac{1}{n-1}\left[-r_{XY}^3 - \frac{1}{2} r_{XY} 1 (-2 r_{XY}{}^2)\right] = 0 \tag{18}$$

$$\mathrm{COV}(r_{XY}, r_{YZ}) = \mathrm{COV}(r_{XY}, r_{XY}) = \mathrm{VAR}(r_{XY}) \tag{19}$$

$$\mathrm{COV}(r_{XZ}, r_{YZ}) = \frac{1}{n-1}\left[r_{XY}(1 - 1^2 - r_{XY}{}^2) - \frac{1}{2} 1 r_{XY}(1 - r_{XY}{}^2 - 1^2 - r_{XY}{}^2)\right]$$

$$= \frac{1}{n-1}\left[r_{XY}(-r_{XY}{}^2) - \frac{1}{2} 1 r_{XY}(-r_{XY}{}^2 - r_{XY}{}^2)\right]$$

$$= \frac{1}{n-1}\left[-r_{XY}{}^3 - \frac{1}{2} r_{XY}(-2 r_{XY}{}^2)\right] = 0 \tag{20}$$

Substituting Equations 16-20 into Equation 2 leads to:

$$\mathrm{VAR}(R_{XY}) = \left(\frac{\partial R_{XY}}{\partial r_{XY}}\right)^2 \mathrm{VAR}(r_{XY}) + \left(\frac{\partial R_{XY}}{\partial r_{XZ}}\right)^2 * 0 + \left(\frac{\partial R_{XY}}{\partial r_{YZ}}\right)^2 \mathrm{VAR}(r_{XY})$$

$$+ 2\left(\frac{\partial R_{XY}}{\partial r_{XY}}\right)\left(\frac{\partial R_{XY}}{\partial r_{XZ}}\right) * 0 + 2\left(\frac{\partial R_{XY}}{\partial r_{XY}}\right)\left(\frac{\partial R_{XY}}{\partial r_{YZ}}\right)\mathrm{VAR}(r_{XY}) + 2\left(\frac{\partial R_{XY}}{\partial r_{XZ}}\right)\left(\frac{\partial R_{XY}}{\partial r_{YZ}}\right) * 0$$

$$= \left(\frac{\partial R_{XY}}{\partial r_{XY}}\right)^2 \mathrm{VAR}(r_{XY}) + \left(\frac{\partial R_{XY}}{\partial r_{YZ}}\right)^2 \mathrm{VAR}(r_{XY}) + 2\left(\frac{\partial R_{XY}}{\partial r_{XY}}\right)\left(\frac{\partial R_{XY}}{\partial r_{YZ}}\right)\mathrm{VAR}(r_{XY})$$

$$= \left(\frac{\partial R_{XY}}{\partial r_{XY}} + \frac{\partial R_{XY}}{\partial r_{YZ}}\right)^2 \mathrm{VAR}(r_{XY}) \tag{21}$$

Based on Equations 3 and 5, the sum of the two partial derivatives in Equation 21 is:

$$\frac{\partial R_{XY}}{\partial r_{XY}} + \frac{\partial R_{XY}}{\partial r_{YZ}} = \frac{1}{\sqrt{1 + (k^2 - 1)r_{XZ}^2}\sqrt{1 + (k^2 - 1)r_{YZ}^2}}$$

$$+ \frac{(k^2 - 1)}{\sqrt{1 + (k^2 - 1)r_{XZ}^2}\left(\sqrt{1 + (k^2 - 1)r_{YZ}^2}\right)^3}[r_{XZ} - r_{XY}r_{YZ}]$$

$$= \frac{1}{\sqrt{1 + (k^2 - 1)1^2}\sqrt{1 + (k^2 - 1)r_{XY}^2}}$$

$$+ \frac{(k^2 - 1)}{\sqrt{1 + (k^2 - 1)1^2}\left(\sqrt{1 + (k^2 - 1)r_{XY}^2}\right)^3}[1 - r_{XY}^2]$$

$$= \frac{1}{\sqrt{k^2}} \cdot \frac{[1 + (k^2 - 1)r_{XY}^2] + (k^2 - 1)(1 - r_{XY}^2)}{\left(\sqrt{1 + (k^2 - 1)r_{XY}^2}\right)^3}$$

$$= \frac{1}{k} \cdot \frac{1 + (k^2 - 1)(r_{XY}^2 + 1 - r_{XY}^2)}{\left(\sqrt{1 + (k^2 - 1)r_{XY}^2}\right)^3}$$

$$= \frac{k^2}{k} \cdot \frac{1}{\left(\sqrt{1 + (k^2 - 1)r_{XY}^2}\right)^3} = \frac{k}{\left(\sqrt{1 + (k^2 - 1)r_{XY}^2}\right)^3} \qquad (22)$$

Substituting Equation 22 and the value for $\mathrm{VAR}(r_{XY})$ from Equation 6 into Equation 21 leads to:

$$\mathrm{VAR}(R_{XY}) = \left(\frac{\partial R_{XY}}{\partial r_{XY}} + \frac{\partial R_{XY}}{\partial r_{YZ}}\right)^2 \mathrm{VAR}(r_{XY}) = \left(\frac{k}{\left(\sqrt{1 + (k^2 - 1)r_{XY}^2}\right)^3}\right)^2 \frac{(1 - r_{XY}^2)^2}{n - 1}$$

$$= \frac{k^2(1 - r_{XY}^2)^2}{(n - 1)(1 + (k^2 - 1)r_{XY}^2)^3} \qquad (23)$$

which is the same as the value for the variance of correlations corrected for DRR presented by Bobko & Rieck (1980).

Appendix B:
The construction of three random variables with a predetermined distribution for one of them and a given covariance matrix

The construction of three random variables X, Y, and Z, with a given covariance matrix $COV = \begin{pmatrix} 1 & \rho_{XY} & \rho_{XZ} \\ \rho_{XY} & 1 & \rho_{YZ} \\ \rho_{XZ} & \rho_{YZ} & 1 \end{pmatrix}$ and a predetermined distribution for one of the random variables (in the present context, Z, the variable on which selection was based) is as follows (see Chan & Chan, 2004, for the bivariate case):

1. Generate Z from the desired distribution (normal, log-normal, or uniform) with the desired parameters.

2. Define $X = cZ + e$, where $c = \rho_{XZ}$ and e is independently generated from a normal distribution $N(0, \sigma_e^2)$, with $\sigma_e^2 = 1 - \rho_{XZ}^2$.

3. Define $Y = aX + bZ + d$, where $a = \frac{\rho_{XY} - \rho_{XZ}\rho_{YZ}}{1 - \rho_{XZ}^2}$, $b = \frac{\rho_{YZ} - \rho_{XY}\rho_{XZ}}{1 - \rho_{XZ}^2}$, and d is independently generated from a normal distribution $N(0, \sigma_d^2)$, with

$$\sigma_d^2 = 1 - \left(\frac{\rho_{XY} - \rho_{XZ}\rho_{YZ}}{1 - \rho_{XZ}^2}\right)^2 - \left(\frac{\rho_{YZ} - \rho_{XY}\rho_{XZ}}{1 - \rho_{XZ}^2}\right)^2 - 2\,\frac{\rho_{XY} - \rho_{XZ}\rho_{YZ}}{1 - \rho_{XZ}^2} \cdot \frac{\rho_{YZ} - \rho_{XY}\rho_{XZ}}{1 - \rho_{XZ}^2} \cdot \rho_{XZ}.$$

It should be noted that when Z is normally distributed, the triplet (X,Y,Z) has a multinormal distribution (Hogg, McKean, & Craig, 2012) and the assumptions of linearity and homoscedasticity are valid. However, when Z is skewed or uniformly distributed, the common distribution of the triplet (X,Y,Z), given the way we defined X and Y above, is unknown.

Appendix C:
Results for nonnormal data

Table 6 - Mean of correlations corrected for IRR ($\overline{R_{XY}}$) and percentage bias ($Bias_R$) in 81 simulation conditions for log-normal data

		Consistent and less sizable (CL): ρ_{XZ}=.20, ρ_{YZ}=.30					
		$\rho_{XY} = .20$		$\rho_{XY} = .50$		$\rho_{XY} = .80$	
n_r	π	$\overline{R_{XY}}$	$Bias_R$	$\overline{R_{XY}}$	$Bias_R$	$\overline{R_{XY}}$	$Bias_R$
20	0.1	.191	-4.3	.492	-1.7	.795	-0.6
	0.3	.195	-2.3	.492	-1.7	.793	-0.8
	0.5	.204	1.9	.495	-1.1	.793	-0.9
60	0.1	.196	-1.8	.496	-0.7	.798	-0.3
	0.3	.198	-1.0	.498	-0.3	.799	-0.1
	0.5	.197	-1.7	.497	-0.7	.798	-0.3
100	0.1	.197	-1.5	.497	-0.6	.798	-0.2
	0.3	.198	-0.8	.499	-0.2	.800	-0.1
	0.5	.197	-1.3	.498	-0.4	.799	-0.1
		Inconsistent (IC): ρ_{XZ}=.20, ρ_{YZ}=.60					
		$\rho_{XY} = .20$		$\rho_{XY} = .50$		$\rho_{XY} = .80$	
n_r	π	$\overline{R_{XY}}$	$Bias_R$	$\overline{R_{XY}}$	$Bias_R$	$\overline{R_{XY}}$	$Bias_R$
20	0.1	.192	-4.2	.493	-1.3	.798	-0.2
	0.3	.191	-4.4	.492	-1.6	.800	-0.0
	0.5	.199	-0.3	.497	-0.7	.802	0.2
60	0.1	.197	-1.7	.498	-0.5	.800	0.0
	0.3	.196	-1.9	.500	-0.0	.804	0.5
	0.5	.193	-3.7	.497	-0.6	.803	0.4
100	0.1	.196	-2.1	.497	-0.6	.799	-0.1
	0.3	.197	-1.6	.499	-0.2	.802	0.3
	0.5	.195	-2.4	.499	-0.2	.804	0.5
		Consistent and more sizable (CM): ρ_{XZ}=.80, ρ_{YZ}=.60					
		$\rho_{XY} = .20$		$\rho_{XY} = .50$		$\rho_{XY} = .80$	
n_r	π	$\overline{R_{XY}}$	$Bias_R$	$\overline{R_{XY}}$	$Bias_R$	$\overline{R_{XY}}$	$Bias_R$
20	0.1	.170	**-14.9**	.479	-4.3	.794	-0.7
	0.3	.135	**-32.5**	.457	-8.6	.784	-2.0
	0.5	.120	**-39.9**	.451	-9.8	.781	-2.4
60	0.1	.183	-8.3	.490	-2.1	.797	-0.3
	0.3	.156	**-21.9**	.475	-5.1	.793	-0.9
	0.5	.144	**-27.9**	.466	-6.8	.789	-1.4
100	0.1	.188	-6.0	.492	-1.6	.797	-0.4
	0.3	.170	**-15.2**	.483	-3.5	.795	-0.6
	0.5	.155	**-22.5**	.474	-5.3	.792	-1.0

Note. ρ_{XY}, ρ_{XZ}, and ρ_{YZ} are the population correlations between variables X and Y, X and Z, and Y and Z, respectively. n_r is the restricted sample size. π is the selection ratio. $\overline{R_{XY}}$ is the mean of the correlations corrected for IRR. $Bias_R$ is the percentage bias of $\overline{R_{XY}}$. $Bias_R$s that are not within ±10% are shown in bold.

Table 7 - Mean standard errors ($\overline{SE_F}, \overline{SE_B}$) and percentage biases ($Bias_{SE_F}, Bias_{SE_B}$) of two standard-error estimates for correlations corrected for IRR in 81 simulation conditions for log-normal data

		Consistent and less sizable (CL): ρ_{XZ}=.20, ρ_{YZ}=.30											
		ρ_{XY} = .20				ρ_{XY} = .50				ρ_{XY} = .80			
n_r	π	$\overline{SE_F}$	$\overline{SE_B}$	$Bias_{SE_F}$	$Bias_{SE_B}$	$\overline{SE_F}$	$\overline{SE_B}$	$Bias_{SE_F}$	$Bias_{SE_B}$	$\overline{SE_F}$	$\overline{SE_B}$	$Bias_{SE_F}$	$Bias_{SE_B}$
20	0.1	0.20	0.21	-5.7	-4.5	0.17	0.17	-4.5	-2.8	0.09	0.09	-3.0	0.2
	0.3	0.21	0.21	-7.6	-7.1	0.17	0.17	-7.0	-6.7	0.09	0.09	-5.2	-4.0
	0.5	0.21	0.21	-9.0	-8.9	0.17	0.17	-7.4	-7.3	0.09	0.09	-4.8	-3.5
60	0.1	0.12	0.12	-3.3	-4.6	0.10	0.09	-1.9	-3.2	0.05	0.05	-0.9	-1.7
	0.3	0.12	0.12	-3.7	-3.5	0.10	0.10	-3.2	-3.3	0.05	0.05	-2.6	-2.8
	0.5	0.12	0.12	-4.8	-4.7	0.10	0.10	-3.6	-4.2	0.05	0.05	-2.1	-2.8
100	0.1	0.09	0.09	-2.1	-2.6	0.07	0.07	-2.7	-3.3	0.04	0.04	-3.0	-3.5
	0.3	0.09	0.09	-5.3	-5.1	0.07	0.07	-3.4	-3.7	0.04	0.04	-0.8	-1.0
	0.5	0.09	0.10	-2.5	-1.2	0.07	0.08	-1.6	-1.1	0.04	0.04	-1.1	-1.1
		Inconsistent (IC): ρ_{XZ}=.20, ρ_{YZ}=.60											
		ρ_{XY} = .20				ρ_{XY} = .50				ρ_{XY} = .80			
n_r	π	$\overline{SE_F}$	$\overline{SE_B}$	$Bias_{SE_F}$	$Bias_{SE_B}$	$\overline{SE_F}$	$\overline{SE_B}$	$Bias_{SE_F}$	$Bias_{SE_B}$	$\overline{SE_F}$	$\overline{SE_B}$	$Bias_{SE_F}$	$Bias_{SE_B}$
20	0.1	0.19	0.20	-4.7	-1.8	0.15	0.16	-2.1	1.7	0.07	0.08	-2.2	6.2
	0.3	0.20	0.20	-8.2	-6.2	0.16	0.16	-6.0	-4.6	0.08	0.08	-5.1	-1.1
	0.5	0.20	0.21	-9.1	-7.9	0.16	0.17	-6.8	-6.4	0.08	0.08	-6.9	-4.3
60	0.1	0.11	0.11	-2.3	-2.3	0.09	0.09	0.3	-0.5	0.04	0.04	-1.5	-0.6
	0.3	0.12	0.12	-5.3	-3.2	0.09	0.09	-1.9	-1.5	0.04	0.04	-8.0	-4.9
	0.5	0.12	0.12	-8.1	-6.0	0.09	0.09	-3.5	-4.0	0.04	0.05	-7.6	-5.3
100	0.1	0.09	0.09	-3.5	-3.0	0.07	0.07	-2.3	-2.7	0.03	0.03	-3.3	-1.9
	0.3	0.09	0.09	-7.9	-5.5	0.07	0.07	-2.9	-2.8	0.03	0.03	-6.2	-2.4
	0.5	0.09	0.10	-5.5	-1.8	0.07	0.07	-0.8	-0.3	0.03	0.04	**-11.2**	-7.4
		Consistent and more sizable (CM): ρ_{XZ}=.80, ρ_{YZ}=.60											
		ρ_{XY} = .20				ρ_{XY} = .50				ρ_{XY} = .80			
n_r	π	$\overline{SE_F}$	$\overline{SE_B}$	$Bias_{SE_F}$	$Bias_{SE_B}$	$\overline{SE_F}$	$\overline{SE_B}$	$Bias_{SE_F}$	$Bias_{SE_B}$	$\overline{SE_F}$	$\overline{SE_B}$	$Bias_{SE_F}$	$Bias_{SE_B}$
20	0.1	0.20	0.20	**-18.3**	**-16.6**	0.16	0.17	**-13.6**	**-10.7**	0.08	0.08	-9.0	-3.0
	0.3	0.20	0.24	**-30.8**	**-18.7**	0.17	0.19	**-22.1**	**-13.4**	0.09	0.10	**-14.6**	-8.2
	0.5	0.20	0.25	**-34.8**	**-19.0**	0.17	0.20	**-24.0**	**-12.8**	0.09	0.10	**-13.3**	-5.2
60	0.1	0.12	0.12	**-20.2**	**-13.7**	0.09	0.09	**-13.5**	-8.3	0.04	0.04	-6.2	-1.4
	0.3	0.12	0.16	**-38.5**	**-19.1**	0.09	0.11	**-29.0**	**-14.5**	0.05	0.05	**-19.8**	-9.4
	0.5	0.12	0.17	**-43.6**	**-18.5**	0.10	0.13	**-33.2**	**-14.4**	0.05	0.06	**-22.8**	**-10.4**
100	0.1	0.09	0.10	**-21.4**	**-11.7**	0.07	0.07	**-15.4**	-8.2	0.03	0.03	**-12.4**	-7.2
	0.3	0.09	0.13	**-40.0**	**-15.4**	0.07	0.09	**-32.0**	**-13.3**	0.03	0.04	**-23.4**	**-10.4**
	0.5	0.09	0.14	**-47.9**	**-19.2**	0.07	0.10	**-36.8**	**-14.1**	0.04	0.04	**-24.5**	-8.1

Note. ρ_{XY}, ρ_{XZ}, and ρ_{YZ} are the population correlations between variables X and Y, X and Z, and Y and Z, respectively. n_r is the restricted sample size. π is the selection ratio. $\overline{SE_F}$ and $\overline{SE_B}$ are the means of the formula and the bootstrap standard errors, respectively, across the 1,000 replications. $Bias_{SE_F}$ and $Bias_{SE_B}$ are the percentage of bias of $\overline{SE_F}$ and $\overline{SE_B}$, respectively. $Bias_{SE_F}$s and $Bias_{SE_B}$s that are not within ±10% are shown in bold.

Table 8 - Coverage probabilities of three confidence intervals for correlations corrected for IRR in 81 simulation conditions for log-normal data

		Consistent and less sizable (CL): ρ_{XZ}=.20, ρ_{YZ}=.30								
		$\rho_{XY} = .20$			$\rho_{XY} = .50$			$\rho_{XY} = .80$		
n_r	π	FSI	BSI	BPI	FSI	BSI	BPI	FSI	BSI	BPI
20	0.1	**0.90**	**0.90**	0.92	**0.90**	**0.91**	0.93	**0.90**	**0.91**	0.92
	0.3	**0.89**	**0.90**	0.93	**0.90**	0.92	0.93	**0.91**	**0.91**	0.93
	0.5	**0.90**	**0.87**	**0.90**	**0.90**	**0.89**	0.92	**0.89**	**0.90**	**0.91**
60	0.1	0.92	0.92	0.93	0.93	0.93	0.93	0.93	0.93	0.94
	0.3	0.92	0.93	0.94	0.93	0.93	0.94	0.92	0.92	0.94
	0.5	0.92	0.93	0.94	0.93	0.93	0.94	0.93	0.93	0.94
100	0.1	0.94	0.93	0.94	0.94	0.93	0.93	0.94	0.94	0.94
	0.3	0.93	0.93	0.94	0.93	0.93	0.94	0.94	0.94	0.95
	0.5	0.93	0.93	0.94	0.92	0.93	0.94	0.93	0.93	0.94
		Inconsistent (IC): ρ_{XZ}=.20, ρ_{YZ}=.60								
		$\rho_{XY} = .20$			$\rho_{XY} = .50$			$\rho_{XY} = .80$		
n_r	π	FSI	BSI	BPI	FSI	BSI	BPI	FSI	BSI	BPI
20	0.1	**0.90**	**0.90**	0.92	0.92	**0.91**	0.93	**0.91**	0.93	0.94
	0.3	**0.89**	**0.90**	0.92	**0.91**	0.92	0.93	**0.89**	**0.90**	0.92
	0.5	**0.91**	**0.90**	0.92	**0.90**	**0.90**	0.92	**0.87**	**0.89**	**0.91**
60	0.1	0.93	0.94	0.94	0.93	0.93	0.95	0.93	0.93	0.94
	0.3	0.93	0.93	0.94	0.93	0.93	0.94	**0.89**	**0.90**	0.92
	0.5	**0.91**	0.92	0.94	0.93	0.94	0.94	**0.89**	**0.89**	**0.91**
100	0.1	0.93	0.93	0.94	0.93	0.94	0.93	0.93	0.94	0.94
	0.3	0.92	0.93	0.94	0.94	0.93	0.94	**0.91**	0.92	0.94
	0.5	0.92	0.93	0.94	0.93	0.93	0.93	**0.90**	**0.91**	0.93
		Consistent and more sizable (CM): ρ_{XZ}=.80, ρ_{YZ}=.60								
		$\rho_{XY} = .20$			$\rho_{XY} = .50$			$\rho_{XY} = .80$		
n_r	π	FSI	BSI	BPI	FSI	BSI	BPI	FSI	BSI	BPI
20	0.1	**0.87**	**0.87**	**0.89**	**0.87**	**0.89**	0.92	**0.88**	0.92	0.92
	0.3	**0.77**	**0.84**	**0.89**	**0.84**	**0.88**	**0.91**	**0.86**	**0.91**	0.92
	0.5	**0.74**	**0.82**	**0.85**	**0.83**	**0.88**	**0.91**	**0.87**	0.93	0.93
60	0.1	**0.88**	**0.89**	0.92	**0.90**	0.92	0.93	**0.91**	0.95	0.94
	0.3	**0.76**	**0.85**	**0.87**	**0.83**	**0.89**	**0.90**	**0.88**	0.93	0.93
	0.5	**0.71**	**0.83**	**0.84**	**0.80**	**0.89**	**0.90**	**0.86**	0.93	0.92
100	0.1	**0.88**	**0.91**	0.93	**0.90**	0.92	0.94	**0.90**	0.93	0.94
	0.3	**0.75**	**0.87**	**0.89**	**0.83**	**0.91**	**0.91**	**0.87**	0.93	0.93
	0.5	**0.67**	**0.83**	**0.83**	**0.77**	**0.89**	**0.88**	**0.86**	0.94	0.93

Note. ρ_{XY}, ρ_{XZ}, and ρ_{YZ} are the population correlations between variables X and Y, X and Z, and Y and Z, respectively. n_r is the restricted sample size. π is the selection ratio. FSI is the 95% formula standard interval. BSI is the 95% bootstrap standard interval. BPI is the 95% bootstrap percentile interval. Coverage probabilities that are below .92 are shown in bold.

Table 9 - Mean width of three confidence intervals for correlations corrected for IRR in 81 simulation conditions for log-normal data

		Consistent and less sizable (CL): ρ_{XZ}=.20, ρ_{YZ}=.30								
		$\rho_{XY} = .20$			$\rho_{XY} = .50$			$\rho_{XY} = .80$		
n_r	π	FSI	BSI	BPI	FSI	BSI	BPI	FSI	BSI	BPI
20	0.1	0.794	0.808	0.803	0.633	0.659	0.655	0.312	0.344	0.339
	0.3	0.801	0.811	0.805	0.639	0.657	0.653	0.318	0.345	0.340
	0.5	0.804	0.811	0.804	0.641	0.658	0.653	0.322	0.348	0.344
60	0.1	0.466	0.460	0.459	0.368	0.366	0.364	0.178	0.180	0.180
	0.3	0.471	0.473	0.471	0.371	0.373	0.372	0.179	0.183	0.182
	0.5	0.475	0.477	0.475	0.374	0.375	0.374	0.182	0.184	0.184
100	0.1	0.362	0.360	0.360	0.285	0.285	0.284	0.138	0.139	0.138
	0.3	0.366	0.367	0.366	0.288	0.288	0.288	0.138	0.140	0.140
	0.5	0.369	0.374	0.373	0.290	0.293	0.292	0.140	0.142	0.142
		Inconsistent (IC): ρ_{XZ}=.20, ρ_{YZ}=.60								
		$\rho_{XY} = .20$			$\rho_{XY} = .50$			$\rho_{XY} = .80$		
n_r	π	FSI	BSI	BPI	FSI	BSI	BPI	FSI	BSI	BPI
20	0.1	0.747	0.776	0.774	0.587	0.624	0.622	0.271	0.309	0.305
	0.3	0.778	0.801	0.796	0.615	0.640	0.637	0.289	0.321	0.317
	0.5	0.792	0.808	0.801	0.626	0.645	0.641	0.297	0.326	0.321
60	0.1	0.430	0.431	0.430	0.333	0.333	0.333	0.150	0.153	0.153
	0.3	0.450	0.461	0.460	0.350	0.354	0.353	0.159	0.168	0.167
	0.5	0.462	0.474	0.473	0.362	0.363	0.362	0.167	0.175	0.174
100	0.1	0.332	0.334	0.334	0.257	0.258	0.257	0.115	0.118	0.118
	0.3	0.348	0.358	0.356	0.270	0.272	0.272	0.123	0.130	0.129
	0.5	0.358	0.373	0.371	0.279	0.282	0.281	0.129	0.136	0.135
		Consistent and more sizable (CM): ρ_{XZ}=.80, ρ_{YZ}=.60								
		$\rho_{XY} = .20$			$\rho_{XY} = .50$			$\rho_{XY} = .80$		
n_r	π	FSI	BSI	BPI	FSI	BSI	BPI	FSI	BSI	BPI
20	0.1	0.773	0.795	0.784	0.608	0.646	0.639	0.288	0.329	0.324
	0.3	0.782	0.930	0.904	0.642	0.739	0.725	0.319	0.373	0.366
	0.5	0.786	0.987	0.952	0.655	0.778	0.761	0.330	0.391	0.383
60	0.1	0.449	0.487	0.481	0.344	0.369	0.366	0.159	0.170	0.170
	0.3	0.459	0.606	0.590	0.363	0.444	0.437	0.172	0.201	0.199
	0.5	0.465	0.675	0.651	0.375	0.489	0.479	0.182	0.218	0.216
100	0.1	0.348	0.392	0.388	0.265	0.290	0.288	0.122	0.131	0.130
	0.3	0.357	0.504	0.492	0.279	0.358	0.353	0.131	0.157	0.155
	0.5	0.362	0.563	0.545	0.288	0.396	0.388	0.138	0.171	0.169

Note. ρ_{XY}, ρ_{XZ}, and ρ_{YZ} are the population correlations between variables X and Y, X and Z, and Y and Z, respectively. n_r is the restricted sample size. π is the selection ratio. FSI is the 95% formula standard interval. BSI is the 95% bootstrap standard interval. BPI is the 95% bootstrap percentile interval.

Table 10 - Mean of correlations corrected for IRR ($\overline{R_{XY}}$) and percentage bias ($Bias_R$) in 81 simulation conditions for (-1) log-normal data*

		Consistent and less sizable (CL): ρ_{XZ}=.20, ρ_{YZ}=.30					
		$\rho_{XY} = .20$		$\rho_{XY} = .50$		$\rho_{XY} = .80$	
n_r	π	$\overline{R_{XY}}$	$Bias_R$	$\overline{R_{XY}}$	$Bias_R$	$\overline{R_{XY}}$	$Bias_R$
20	0.1	.142	**-28.9**	.344	**-31.3**	.614	**-23.2**
	0.3	.080	**-60.0**	.350	**-29.9**	.686	**-14.2**
	0.5	.127	**-36.5**	.408	**-18.4**	.722	-9.8
60	0.1	.135	**-32.6**	.393	**-21.4**	.684	**-14.4**
	0.3	.139	**-30.6**	.429	**-14.3**	.742	-7.2
	0.5	.121	**-39.6**	.425	**-15.0**	.758	-5.3
100	0.1	.121	**-39.5**	.388	**-22.3**	.689	**-13.9**
	0.3	.141	**-29.6**	.437	**-12.6**	.756	-5.5
	0.5	.158	**-20.9**	.458	-8.3	.776	-3.1
		Inconsistent (IC): ρ_{XZ}=.20, ρ_{YZ}=.60					
		$\rho_{XY} = .20$		$\rho_{XY} = .50$		$\rho_{XY} = .80$	
n_r	π	$\overline{R_{XY}}$	$Bias_R$	$\overline{R_{XY}}$	$Bias_R$	$\overline{R_{XY}}$	$Bias_R$
20	0.1	.111	**-44.4**	.348	**-30.5**	.699	**-12.6**
	0.3	.037	**-81.3**	.367	**-26.5**	.771	-3.6
	0.5	.089	**-55.5**	.415	**-16.9**	.790	-1.2
60	0.1	.097	**-51.3**	.397	**-20.5**	.769	-3.9
	0.3	.099	**-50.3**	.444	**-11.2**	.802	0.3
	0.5	.090	**-55.0**	.429	**-14.2**	.804	0.4
100	0.1	.084	**-57.9**	.398	**-20.5**	.775	-3.2
	0.3	.110	**-44.8**	.447	**-10.5**	.809	1.1
	0.5	.129	**-35.4**	.457	-8.7	.803	0.3
		Consistent and more sizable (CM): ρ_{XZ}=.80, ρ_{YZ}=.60					
		$\rho_{XY} = .20$		$\rho_{XY} = .50$		$\rho_{XY} = .80$	
n_r	π	$\overline{R_{XY}}$	$Bias_R$	$\overline{R_{XY}}$	$Bias_R$	$\overline{R_{XY}}$	$Bias_R$
20	0.1	-.39	**-296.0**	.063	**-87.4**	.488	**-39.0**
	0.3	-.42	**-309.0**	.012	**-97.5**	.541	**-32.4**
	0.5	-.31	**-255.0**	.116	**-76.7**	.599	**-25.2**
60	0.1	-.39	**-293.0**	.075	**-85.0**	.557	**-30.3**
	0.3	-.31	**-253.0**	.147	**-70.5**	.652	**-18.5**
	0.5	-.16	**-182.0**	.213	**-57.5**	.666	**-16.8**
100	0.1	-.41	**-303.0**	.053	**-89.4**	.553	**-30.8**
	0.3	-.24	**-221.0**	.189	**-62.2**	.671	**-16.2**
	0.5	-.06	**-129.0**	.304	**-39.2**	.709	**-11.4**

Note. ρ_{XY}, ρ_{XZ}, and ρ_{YZ} are the population correlations between variables X and Y, X and Z, and Y and Z, respectively. n_r is the restricted sample size. π is the selection ratio. $\overline{R_{XY}}$ is the mean of the correlations corrected for IRR. $Bias_R$ is the percentage bias of $\overline{R_{XY}}$. $Bias_R$s that are not within ±10% are shown in bold.

Table 11 - Mean standard errors ($\overline{SE_F}, \overline{SE_B}$) and percentage biases ($Bias_{SE_F}, Bias_{SE_B}$) of two standard-error estimates for correlations corrected for IRR in 81 simulation conditions for (-1) log-normal data*

		Consistent and less sizable (CL): ρ_{XZ}=.20, ρ_{YZ}=.30											
		ρ_{XY} = .20				ρ_{XY} = .50				ρ_{XY} = .80			
n_r	π	$\overline{SE_F}$	$\overline{SE_B}$	$Bias_{SE_F}$	$Bias_{SE_B}$	$\overline{SE_F}$	$\overline{SE_B}$	$Bias_{SE_F}$	$Bias_{SE_B}$	$\overline{SE_F}$	$\overline{SE_B}$	$Bias_{SE_F}$	$Bias_{SE_B}$
20	0.1	2.57	0.80	**205.2**	-5.1	2.40	0.75	**202.4**	-5.1	2.20	0.61	**245.1**	-4.3
	0.3	1.36	0.66	**97.1**	-4.1	1.29	0.61	**103.2**	-4.3	1.21	0.45	**171.0**	1.3
	0.5	0.98	0.53	**82.3**	-1.2	0.87	0.48	**80.4**	-0.5	0.66	0.33	**100.5**	0.8
60	0.1	1.78	0.74	**135.4**	-1.5	1.62	0.69	**132.7**	-0.7	1.46	0.53	**195.1**	6.1
	0.3	0.96	0.56	**76.3**	2.6	0.87	0.50	**79.1**	3.7	0.61	0.34	**92.5**	7.1
	0.5	0.58	0.41	**49.8**	5.2	0.52	0.36	**54.1**	6.0	0.34	0.22	**71.1**	**10.1**
100	0.1	1.50	0.71	**114.3**	1.1	1.41	0.65	**120.4**	2.0	1.10	0.48	**137.1**	4.1
	0.3	0.77	0.50	**62.7**	5.7	0.69	0.44	**67.7**	8.3	0.45	0.28	**78.9**	**12.5**
	0.5	0.45	0.34	**41.9**	8.3	0.38	0.29	**43.5**	**10.8**	0.22	0.17	**54.6**	**18.0**
		Inconsistent (IC): ρ_{XZ}=.20, ρ_{YZ}=.60											
		ρ_{XY} = .20				ρ_{XY} = .50				ρ_{XY} = .80			
n_r	π	$\overline{SE_F}$	$\overline{SE_B}$	$Bias_{SE_F}$	$Bias_{SE_B}$	$\overline{SE_F}$	$\overline{SE_B}$	$Bias_{SE_F}$	$Bias_{SE_B}$	$\overline{SE_F}$	$\overline{SE_B}$	$Bias_{SE_F}$	$Bias_{SE_B}$
20	0.1	2.56	0.80	**202.9**	-5.2	2.40	0.75	**202.6**	-5.4	2.11	0.54	**278.3**	-4.0
	0.3	1.37	0.66	**98.8**	-4.0	1.32	0.61	**110.1**	-3.5	0.99	0.38	**160.8**	-0.3
	0.5	0.94	0.53	**70.5**	-3.4	0.88	0.48	**81.4**	-1.8	0.56	0.28	**99.3**	-2.7
60	0.1	1.77	0.75	**131.4**	-2.1	1.62	0.69	**133.8**	-1.3	1.27	0.45	**206.1**	7.6
	0.3	0.98	0.57	**77.2**	1.9	0.87	0.50	**81.5**	4.3	0.53	0.28	**94.4**	0.2
	0.5	0.58	0.41	**40.0**	-0.1	0.53	0.36	**49.3**	1.5	0.29	0.19	**51.5**	-2.0
100	0.1	1.51	0.71	**113.3**	0.5	1.41	0.65	**122.0**	1.8	1.00	0.40	**165.5**	6.5
	0.3	0.79	0.51	**62.3**	4.1	0.69	0.44	**64.8**	6.0	0.40	0.23	**74.9**	1.4
	0.5	0.46	0.35	**32.1**	1.2	0.39	0.30	**35.2**	2.3	0.22	0.15	**35.8**	-5.4
		Consistent and more sizable (CM): ρ_{XZ}=.80, ρ_{YZ}=.60											
		ρ_{XY} = .20				ρ_{XY} = .50				ρ_{XY} = .80			
n_r	π	$\overline{SE_F}$	$\overline{SE_B}$	$Bias_{SE_F}$	$Bias_{SE_B}$	$\overline{SE_F}$	$\overline{SE_B}$	$Bias_{SE_F}$	$Bias_{SE_B}$	$\overline{SE_F}$	$\overline{SE_B}$	$Bias_{SE_F}$	$Bias_{SE_B}$
20	0.1	2.32	0.73	**201.0**	-5.3	2.45	0.80	**186.4**	-5.8	2.15	0.69	**195.5**	-5.7
	0.3	1.27	0.58	**103.3**	-8.0	1.38	0.66	**97.6**	-5.5	1.25	0.53	**127.8**	-3.3
	0.5	0.84	0.48	**49.2**	**-13.9**	0.86	0.53	**47.9**	-8.5	0.73	0.41	**68.8**	-6.6
60	0.1	1.72	0.66	**147.4**	-4.8	1.77	0.75	**131.9**	-2.2	1.57	0.61	**161.5**	1.8
	0.3	0.92	0.51	**62.3**	**-10.2**	0.95	0.56	**66.6**	-1.5	0.71	0.41	**84.9**	8.2
	0.5	0.59	0.41	**30.0**	**-11.2**	0.58	0.41	**31.2**	-5.9	0.42	0.28	**43.4**	-2.1
100	0.1	1.36	0.63	**111.7**	-2.8	1.49	0.71	**108.7**	-0.7	1.28	0.57	**127.4**	0.8
	0.3	0.74	0.47	**39.4**	**-12.4**	0.77	0.51	**50.4**	-1.4	0.55	0.36	**70.7**	9.8
	0.5	0.46	0.36	**12.2**	**-11.4**	0.43	0.35	**14.8**	-7.7	0.29	0.22	**29.2**	-2.0

Note. ρ_{XY}, ρ_{XZ}, and ρ_{YZ} are the population correlations between variables X and Y, X and Z, and Y and Z, respectively. n_r is the restricted sample size. π is the selection ratio. $\overline{SE_F}$ and $\overline{SE_B}$ are the means of the formula and the bootstrap standard errors, respectively, across the 1,000 replications. $Bias_{SE_F}$ and $Bias_{SE_B}$ are the percentage of bias of $\overline{SE_F}$ and $\overline{SE_B}$, respectively. $Bias_{SE_F}$s and $Bias_{SE_B}$s that are not within ±10% are shown in bold.

Table 12 - Coverage probabilities of three confidence intervals for correlations corrected for IRR in 81 simulation conditions for (-1) log-normal data*

		Consistent and less sizable (CL): ρ_{XZ}=.20, ρ_{YZ}=.30								
		$\rho_{XY}=.20$			$\rho_{XY}=.50$			$\rho_{XY}=.80$		
n_r	π	FSI	BSI	BPI	FSI	BSI	BPI	FSI	BSI	BPI
20	0.1	**0.42**	0.98	1.00	**0.41**	0.98	0.99	**0.44**	0.92	0.98
	0.3	**0.64**	0.95	1.00	**0.66**	0.96	0.99	**0.67**	0.96	0.98
	0.5	**0.78**	0.94	0.99	**0.78**	0.95	0.99	**0.79**	0.96	0.99
60	0.1	**0.55**	0.98	1.00	**0.55**	0.98	1.00	**0.57**	0.95	0.99
	0.3	**0.78**	0.96	1.00	**0.78**	0.96	0.99	**0.76**	0.98	0.99
	0.5	**0.89**	0.95	1.00	**0.89**	0.97	1.00	**0.90**	0.98	1.00
100	0.1	**0.65**	0.98	1.00	**0.64**	0.98	1.00	**0.63**	0.96	0.99
	0.3	**0.84**	0.97	1.00	**0.83**	0.97	1.00	**0.82**	0.98	0.99
	0.5	**0.91**	0.96	1.00	0.92	0.96	0.99	**0.91**	0.98	1.00
		Inconsistent (IC): ρ_{XZ}=.20, ρ_{YZ}=.60								
		$\rho_{XY}=.20$			$\rho_{XY}=.50$			$\rho_{XY}=.80$		
n_r	π	FSI	BSI	BPI	FSI	BSI	BPI	FSI	BSI	BPI
20	0.1	**0.42**	0.98	1.00	**0.41**	0.98	0.99	**0.42**	0.93	0.97
	0.3	**0.66**	0.95	1.00	**0.67**	0.96	0.99	**0.61**	0.95	0.98
	0.5	**0.78**	0.93	0.99	**0.77**	0.95	0.99	**0.71**	0.94	0.97
60	0.1	**0.55**	0.97	1.00	**0.54**	0.98	1.00	**0.52**	0.96	0.98
	0.3	**0.78**	0.95	1.00	**0.76**	0.96	0.99	**0.66**	0.96	0.99
	0.5	**0.86**	0.93	0.99	**0.88**	0.95	1.00	**0.81**	0.93	0.98
100	0.1	**0.63**	0.98	1.00	**0.64**	0.98	1.00	**0.58**	0.97	0.99
	0.3	**0.83**	0.95	1.00	**0.82**	0.97	1.00	**0.73**	0.96	0.98
	0.5	**0.89**	0.94	0.99	**0.89**	0.95	0.99	**0.81**	0.92	0.97
		Consistent and more sizable (CM): ρ_{XZ}=.80, ρ_{YZ}=.60								
		$\rho_{XY}=.20$			$\rho_{XY}=.50$			$\rho_{XY}=.80$		
n_r	π	FSI	BSI	BPI	FSI	BSI	BPI	FSI	BSI	BPI
20	0.1	**0.39**	**0.84**	0.98	**0.44**	0.93	1.00	**0.47**	**0.90**	0.99
	0.3	**0.53**	**0.68**	0.95	**0.64**	**0.86**	1.00	**0.71**	0.94	0.99
	0.5	**0.62**	**0.66**	**0.88**	**0.75**	**0.83**	0.98	**0.81**	0.93	0.99
60	0.1	**0.49**	**0.78**	0.99	**0.58**	**0.91**	1.00	**0.60**	0.94	0.99
	0.3	**0.65**	**0.69**	0.95	**0.78**	**0.88**	0.99	**0.80**	0.95	0.99
	0.5	**0.77**	**0.73**	**0.86**	**0.84**	**0.86**	0.96	**0.88**	0.96	0.99
100	0.1	**0.54**	**0.75**	0.99	**0.64**	**0.90**	1.00	**0.68**	0.93	1.00
	0.3	**0.69**	**0.68**	0.92	**0.81**	**0.88**	0.99	**0.83**	0.97	0.99
	0.5	**0.82**	**0.79**	**0.87**	**0.84**	**0.87**	0.94	**0.86**	0.94	0.98

Note. ρ_{XY}, ρ_{XZ}, and ρ_{YZ} are the population correlations between variables X and Y, X and Z, and Y and Z, respectively. n_r is the restricted sample size. π is the selection ratio. FSI is the 95% formula standard interval. BSI is the 95% bootstrap standard interval. BPI is the 95% bootstrap percentile interval. Coverage probabilities that are below .92 are shown in bold.

Table 13 - Mean width of three confidence intervals for correlations corrected for IRR in 81 simulation conditions for (-1) log-normal data*

		Consistent and less sizable (CL): ρ_{XZ}=.20, ρ_{YZ}=.30								
		$\rho_{XY}=.20$			$\rho_{XY}=.50$			$\rho_{XY}=.80$		
n_r	π	FSI	BSI	BPI	FSI	BSI	BPI	FSI	BSI	BPI
20	0.1	5.227	3.131	1.972	4.788	2.950	1.944	4.050	2.392	1.827
	0.3	3.828	2.593	1.891	3.513	2.391	1.810	2.720	1.772	1.510
	0.5	2.993	2.084	1.715	2.576	1.882	1.595	1.657	1.303	1.181
60	0.1	4.620	2.917	1.945	3.997	2.703	1.905	3.284	2.062	1.691
	0.3	3.148	2.200	1.761	2.692	1.966	1.637	1.648	1.321	1.200
	0.5	2.015	1.592	1.457	1.731	1.395	1.296	0.989	0.861	0.823
100	0.1	4.320	2.776	1.926	3.885	2.553	1.872	2.685	1.891	1.600
	0.3	2.666	1.971	1.664	2.274	1.735	1.515	1.315	1.109	1.032
	0.5	1.591	1.335	1.275	1.308	1.137	1.098	0.701	0.660	0.643
		Inconsistent (IC): ρ_{XZ}=.20, ρ_{YZ}=.60								
		$\rho_{XY}=.20$			$\rho_{XY}=.50$			$\rho_{XY}=.80$		
n_r	π	FSI	BSI	BPI	FSI	BSI	BPI	FSI	BSI	BPI
20	0.1	5.244	3.143	1.973	4.767	2.936	1.942	3.766	2.097	1.728
	0.3	3.846	2.602	1.894	3.548	2.380	1.806	2.139	1.483	1.321
	0.5	2.927	2.094	1.721	2.577	1.872	1.585	1.363	1.076	0.984
60	0.1	4.579	2.928	1.947	4.002	2.686	1.900	2.697	1.751	1.527
	0.3	3.178	2.217	1.767	2.688	1.957	1.630	1.337	1.078	0.991
	0.5	2.011	1.610	1.467	1.739	1.399	1.295	0.844	0.728	0.689
100	0.1	4.313	2.784	1.929	3.861	2.534	1.867	2.315	1.579	1.405
	0.3	2.697	1.988	1.673	2.249	1.727	1.507	1.091	0.905	0.840
	0.5	1.638	1.371	1.298	1.352	1.161	1.110	0.673	0.595	0.567
		Consistent and more sizable (CM): ρ_{XZ}=.80, ρ_{YZ}=.60								
		$\rho_{XY}=.20$			$\rho_{XY}=.50$			$\rho_{XY}=.80$		
n_r	π	FSI	BSI	BPI	FSI	BSI	BPI	FSI	BSI	BPI
20	0.1	4.681	2.860	1.930	5.047	3.151	1.973	4.237	2.689	1.898
	0.3	3.272	2.262	1.756	3.838	2.591	1.892	3.175	2.088	1.676
	0.5	2.492	1.892	1.588	2.688	2.092	1.720	1.978	1.589	1.394
60	0.1	4.207	2.600	1.879	4.616	2.931	1.949	3.727	2.387	1.815
	0.3	2.818	1.989	1.630	3.023	2.206	1.761	2.014	1.624	1.418
	0.5	2.018	1.587	1.408	1.961	1.623	1.467	1.243	1.110	1.037
100	0.1	3.687	2.449	1.829	4.219	2.786	1.929	3.306	2.228	1.754
	0.3	2.434	1.833	1.546	2.561	1.978	1.666	1.648	1.398	1.262
	0.5	1.643	1.427	1.308	1.499	1.364	1.278	0.904	0.869	0.830

Note. ρ_{XY}, ρ_{XZ}, and ρ_{YZ} are the population correlations between variables X and Y, X and Z, and Y and Z, respectively. n_r is the restricted sample size. π is the selection ratio. FSI is the 95% formula standard interval. BSI is the 95% bootstrap standard interval. BPI is the 95% bootstrap percentile interval.

Table 14 - Mean of correlations corrected for IRR ($\overline{R_{XY}}$) and percentage bias ($Bias_R$) in 81 simulation conditions for uniform data

		Consistent and less sizable (CL): ρ_{XZ}=.20, ρ_{YZ}=.30					
		ρ_{XY} = .20		ρ_{XY} = .50		ρ_{XY} = .80	
n_r	π	$\overline{R_{XY}}$	$Bias_R$	$\overline{R_{XY}}$	$Bias_R$	$\overline{R_{XY}}$	$Bias_R$
20	0.1	.152	**-23.8**	.424	**-15.1**	.729	-8.9
	0.3	.172	**-14.2**	.470	-6.0	.781	-2.4
	0.5	.193	-3.6	.486	-2.8	.788	-1.6
60	0.1	.175	**-12.4**	.461	-7.8	.764	-4.5
	0.3	.173	**-13.5**	.474	-5.2	.786	-1.8
	0.5	.194	-2.9	.493	-1.5	.795	-0.6
100	0.1	.147	**-26.4**	.444	**-11.1**	.765	-4.3
	0.3	.190	-5.2	.490	-1.9	.795	-0.6
	0.5	.197	-1.6	.495	-1.0	.797	-0.4
		Inconsistent (IC): ρ_{XZ}=.20, ρ_{YZ}=.60					
		ρ_{XY} = .20		ρ_{XY} = .50		ρ_{XY} = .80	
n_r	π	$\overline{R_{XY}}$	$Bias_R$	$\overline{R_{XY}}$	$Bias_R$	$\overline{R_{XY}}$	$Bias_R$
20	0.1	.124	**-38.2**	.432	**-13.7**	.789	-1.4
	0.3	.156	**-22.0**	.471	-5.9	.801	0.1
	0.5	.183	-8.3	.485	-2.9	.796	-0.6
60	0.1	.147	**-26.5**	.471	-5.7	.810	1.2
	0.3	.159	**-20.3**	.471	-5.8	.792	-0.9
	0.5	.191	-4.4	.492	-1.6	.796	-0.4
100	0.1	.128	**-35.9**	.451	-9.7	.800	0.0
	0.3	.185	-7.4	.491	-1.7	.800	-0.0
	0.5	.195	-2.5	.495	-1.1	.798	-0.3
		Consistent and more sizable (CM): ρ_{XZ}=.80, ρ_{YZ}=.60					
		ρ_{XY} = .20		ρ_{XY} = .50		ρ_{XY} = .80	
n_r	π	$\overline{R_{XY}}$	$Bias_R$	$\overline{R_{XY}}$	$Bias_R$	$\overline{R_{XY}}$	$Bias_R$
20	0.1	-.27	**-235.0**	.167	**-66.5**	.636	**-20.5**
	0.3	.023	**-88.6**	.354	**-29.3**	.724	-9.5
	0.5	.139	**-30.3**	.435	**-12.9**	.755	-5.6
60	0.1	-.16	**-179.0**	.241	**-51.8**	.672	**-16.1**
	0.3	.126	**-37.1**	.427	**-14.7**	.757	-5.4
	0.5	.183	-8.5	.480	-4.1	.785	-1.8
100	0.1	-.08	**-140.0**	.282	**-43.6**	.693	**-13.4**
	0.3	.154	**-22.9**	.459	-8.2	.776	-2.9
	0.5	.190	-5.1	.488	-2.4	.791	-1.1

Note. ρ_{XY}, ρ_{XZ}, and ρ_{YZ} are the population correlations between variables X and Y, X and Z, and Y and Z, respectively. n_r is the restricted sample size. π is the selection ratio. $\overline{R_{XY}}$ is the mean of the correlations corrected for IRR. $Bias_R$ is the percentage bias of $\overline{R_{XY}}$. $Bias_R$s that are not within ±10% are shown in bold.

Table 15 - Mean standard errors ($\overline{SE_F}$, $\overline{SE_B}$) and percentage biases ($Bias_{SE_F}$, $Bias_{SE_B}$) of two standard-error estimates for correlations corrected for IRR in 81 simulation conditions for uniform data

		Consistent and less sizable (CL): ρ_{XZ}=.20, ρ_{YZ}=.30											
		ρ_{XY} = .20				ρ_{XY} = .50				ρ_{XY} = .80			
n_r	π	$\overline{\mathrm{SE_F}}$	$\overline{\mathrm{SE_B}}$	$\mathrm{Bias_{SE_F}}$	$\mathrm{Bias_{SE_B}}$	$\overline{\mathrm{SE_F}}$	$\overline{\mathrm{SE_B}}$	$\mathrm{Bias_{SE_F}}$	$\mathrm{Bias_{SE_B}}$	$\overline{\mathrm{SE_F}}$	$\overline{\mathrm{SE_B}}$	$\mathrm{Bias_{SE_F}}$	$\mathrm{Bias_{SE_B}}$
20	0.1	1.07	0.62	**75.8**	0.6	1.03	0.56	**94.0**	4.6	0.77	0.40	**115.2**	**10.4**
	0.3	0.43	0.35	**27.4**	4.4	0.36	0.30	**32.1**	9.6	0.21	0.18	**36.2**	**15.5**
	0.5	0.28	0.26	**13.8**	7.1	0.23	0.22	**16.9**	**11.5**	0.12	0.12	**20.5**	**17.3**
60	0.1	0.69	0.48	**63.5**	**12.2**	0.60	0.41	**64.9**	**13.4**	0.39	0.26	**76.6**	**18.4**
	0.3	0.22	0.21	**13.4**	5.7	0.18	0.17	**15.8**	8.4	0.10	0.09	**21.1**	**14.2**
	0.5	0.15	0.15	2.7	0.5	0.12	0.12	4.2	2.7	0.06	0.06	5.5	4.7
100	0.1	0.53	0.40	**46.1**	**10.1**	0.47	0.35	**50.6**	**12.4**	0.29	0.21	**61.9**	**17.9**
	0.3	0.16	0.16	9.5	6.3	0.13	0.12	**11.1**	9.1	0.06	0.06	**14.8**	**13.5**
	0.5	0.11	0.11	-0.3	-1.8	0.09	0.09	0.6	-0.5	0.04	0.04	0.7	-0.1
		Inconsistent (IC): ρ_{XZ}=.20, ρ_{YZ}=.60											
		ρ_{XY} = .20				ρ_{XY} = .50				ρ_{XY} = .80			
n_r	π	$\overline{\mathrm{SE_F}}$	$\overline{\mathrm{SE_B}}$	$\mathrm{Bias_{SE_F}}$	$\mathrm{Bias_{SE_B}}$	$\overline{\mathrm{SE_F}}$	$\overline{\mathrm{SE_B}}$	$\mathrm{Bias_{SE_F}}$	$\mathrm{Bias_{SE_B}}$	$\overline{\mathrm{SE_F}}$	$\overline{\mathrm{SE_B}}$	$\mathrm{Bias_{SE_F}}$	$\mathrm{Bias_{SE_B}}$
20	0.1	1.10	0.62	**77.5**	-0.4	1.03	0.55	**92.3**	3.7	0.66	0.33	**111.8**	5.5
	0.3	0.43	0.36	**17.2**	-3.0	0.37	0.30	**19.3**	-1.3	0.21	0.16	**28.1**	-1.8
	0.5	0.29	0.27	4.3	-2.5	0.24	0.23	7.5	0.4	0.14	0.12	**11.2**	0.4
60	0.1	0.71	0.48	**59.7**	8.8	0.60	0.41	**61.6**	**10.4**	0.35	0.22	**70.5**	5.7
	0.3	0.25	0.23	8.1	-2.4	0.21	0.19	8.3	-2.4	0.12	0.10	**10.0**	-5.8
	0.5	0.17	0.16	0.8	-3.9	0.14	0.13	2.0	-3.0	0.08	0.07	5.5	-0.8
100	0.1	0.54	0.41	**38.8**	4.3	0.47	0.35	**41.0**	4.6	0.27	0.18	**45.4**	-2.4
	0.3	0.19	0.18	4.3	-3.1	0.16	0.15	6.0	-2.0	0.09	0.08	8.1	-2.6
	0.5	0.13	0.13	-3.5	-6.7	0.11	0.11	-1.6	-4.9	0.06	0.06	1.7	-2.4
		Consistent and more sizable (CM): ρ_{XZ}=.80, ρ_{YZ}=.60											
		ρ_{XY} = .20				ρ_{XY} = .50				ρ_{XY} = .80			
n_r	π	$\overline{\mathrm{SE_F}}$	$\overline{\mathrm{SE_B}}$	$\mathrm{Bias_{SE_F}}$	$\mathrm{Bias_{SE_B}}$	$\overline{\mathrm{SE_F}}$	$\overline{\mathrm{SE_B}}$	$\mathrm{Bias_{SE_F}}$	$\mathrm{Bias_{SE_B}}$	$\overline{\mathrm{SE_F}}$	$\overline{\mathrm{SE_B}}$	$\mathrm{Bias_{SE_F}}$	$\mathrm{Bias_{SE_B}}$
20	0.1	1.04	0.56	**67.6**	-9.1	1.11	0.61	**77.2**	-2.3	0.91	0.47	**109.9**	8.4
	0.3	0.45	0.37	**14.4**	-5.6	0.43	0.36	**15.9**	-3.8	0.28	0.23	**24.8**	3.4
	0.5	0.30	0.28	**11.3**	2.0	0.27	0.25	2.6	-3.3	0.17	0.16	4.2	-3.1
60	0.1	0.67	0.46	**31.8**	-9.1	0.66	0.48	**36.5**	-1.5	0.47	0.33	**44.6**	2.1
	0.3	0.25	0.24	6.1	-1.4	0.23	0.22	2.6	-4.6	0.14	0.13	8.0	-1.1
	0.5	0.17	0.15	**12.8**	5.1	0.14	0.14	8.4	4.6	0.08	0.08	6.8	4.3
100	0.1	0.56	0.42	**25.7**	-6.5	0.53	0.41	**24.3**	-3.6	0.36	0.27	**37.8**	1.8
	0.3	0.19	0.18	**10.3**	5.1	0.17	0.17	5.0	1.1	0.10	0.10	6.4	2.0
	0.5	0.13	0.12	7.7	-0.1	0.11	0.11	1.4	-2.6	0.06	0.06	-2.0	-4.4

Note. ρ_{XY}, ρ_{XZ}, and ρ_{YZ} are the population correlations between variables X and Y, X and Z, and Y and Z, respectively. n_r is the restricted sample size. π is the selection ratio. $\overline{\mathrm{SE_F}}$ and $\overline{\mathrm{SE_B}}$ are the means of the formula and the bootstrap standard errors, respectively, across the 1,000 replications. $\mathrm{Bias_{SE_F}}$ and $\mathrm{Bias_{SE_B}}$ are the percentage of bias of $\overline{\mathrm{SE_F}}$ and $\overline{\mathrm{SE_B}}$, respectively. $\mathrm{Bias_{SE_F}}$s and $\mathrm{Bias_{SE_B}}$s that are not within ±10% are shown in bold.

Table 16 - Coverage probabilities of three confidence intervals for correlations corrected for IRR in 81 simulation conditions for uniform data

		Consistent and less sizable (CL): ρ_{XZ}=.20, ρ_{YZ}=.30								
		$\rho_{XY} = .20$			$\rho_{XY} = .50$			$\rho_{XY} = .80$		
n_r	π	FSI	BSI	BPI	FSI	BSI	BPI	FSI	BSI	BPI
20	0.1	**0.72**	0.95	0.99	**0.71**	0.96	0.99	**0.71**	0.97	0.99
	0.3	**0.90**	0.94	0.98	**0.91**	0.95	0.98	**0.89**	0.95	0.97
	0.5	0.94	0.94	0.97	0.94	0.95	0.97	0.93	0.95	0.98
60	0.1	**0.87**	0.97	1.00	**0.87**	0.97	1.00	**0.84**	0.97	0.99
	0.3	0.94	0.95	0.97	0.96	0.96	0.98	0.95	0.97	0.98
	0.5	0.94	0.94	0.95	0.95	0.96	0.95	0.95	0.95	0.96
100	0.1	**0.89**	0.97	1.00	**0.90**	0.97	0.99	**0.90**	0.98	0.99
	0.3	0.96	0.96	0.98	0.96	0.96	0.98	0.96	0.97	0.98
	0.5	0.93	0.93	0.94	0.94	0.94	0.95	0.95	0.95	0.95
		Inconsistent (IC): ρ_{XZ}=.20, ρ_{YZ}=.60								
		$\rho_{XY} = .20$			$\rho_{XY} = .50$			$\rho_{XY} = .80$		
n_r	π	FSI	BSI	BPI	FSI	BSI	BPI	FSI	BSI	BPI
20	0.1	**0.70**	0.95	0.99	**0.71**	0.95	0.99	**0.64**	0.96	0.97
	0.3	**0.87**	**0.90**	0.97	**0.87**	0.92	0.97	**0.81**	**0.91**	0.96
	0.5	0.92	0.92	0.96	**0.91**	0.93	0.96	**0.86**	**0.89**	0.94
60	0.1	**0.86**	0.97	1.00	**0.84**	0.96	0.99	**0.76**	0.94	0.98
	0.3	0.92	0.93	0.96	0.92	0.93	0.97	**0.89**	**0.90**	0.96
	0.5	0.93	0.92	0.95	0.93	0.92	0.94	0.92	0.92	0.95
100	0.1	**0.87**	0.95	0.99	**0.86**	0.95	0.99	**0.80**	0.93	0.98
	0.3	0.93	0.93	0.97	0.92	0.92	0.97	**0.90**	**0.90**	0.95
	0.5	0.92	**0.91**	0.93	0.92	0.92	0.93	0.93	0.92	0.94
		Consistent and more sizable (CM): ρ_{XZ}=.80, ρ_{YZ}=.60								
		$\rho_{XY} = .20$			$\rho_{XY} = .50$			$\rho_{XY} = .80$		
n_r	π	FSI	BSI	BPI	FSI	BSI	BPI	FSI	BSI	BPI
20	0.1	**0.63**	**0.74**	0.96	**0.70**	**0.88**	0.99	**0.75**	0.96	0.98
	0.3	**0.87**	**0.87**	0.93	**0.86**	**0.90**	0.95	**0.87**	0.94	0.96
	0.5	0.92	**0.91**	0.94	**0.90**	**0.90**	0.93	**0.90**	0.93	0.93
60	0.1	**0.75**	**0.76**	0.93	**0.83**	**0.89**	0.99	**0.82**	0.95	0.98
	0.3	0.94	0.92	0.93	0.92	0.92	0.94	0.92	0.94	0.95
	0.5	0.96	0.95	0.96	0.95	0.95	0.95	0.93	0.94	0.95
100	0.1	**0.83**	**0.81**	0.92	**0.85**	**0.89**	0.98	**0.86**	0.95	0.98
	0.3	0.96	0.95	0.95	0.93	0.93	0.95	0.92	0.93	0.95
	0.5	0.95	0.94	0.94	0.94	0.93	0.93	0.93	0.93	0.92

Note. ρ_{XY}, ρ_{XZ}, and ρ_{YZ} are the population correlations between variables X and Y, X and Z, and Y and Z, respectively. n_r is the restricted sample size. π is the selection ratio. FSI is the 95% formula standard interval. BSI is the 95% bootstrap standard interval. BPI is the 95% bootstrap percentile interval. Coverage probabilities that are below .92 are shown in bold.

Table 17 - Mean width of three confidence intervals for correlations corrected for IRR in 81 simulation conditions for uniform data

		Consistent and less sizable (CL): ρ_{XZ}=.20, ρ_{YZ}=.30								
		$\rho_{XY} = .20$			$\rho_{XY} = .50$			$\rho_{XY} = .80$		
n_r	π	FSI	BSI	BPI	FSI	BSI	BPI	FSI	BSI	BPI
20	0.1	3.336	2.409	1.825	3.045	2.185	1.723	1.981	1.551	1.354
	0.3	1.564	1.376	1.289	1.284	1.176	1.112	0.674	0.690	0.661
	0.5	1.072	1.035	1.012	0.859	0.859	0.842	0.439	0.473	0.462
60	0.1	2.449	1.864	1.601	2.028	1.620	1.436	1.157	1.013	0.946
	0.3	0.841	0.810	0.805	0.675	0.664	0.660	0.341	0.351	0.348
	0.5	0.579	0.572	0.569	0.453	0.454	0.452	0.223	0.228	0.227
100	0.1	1.941	1.578	1.435	1.634	1.362	1.260	0.908	0.812	0.773
	0.3	0.613	0.610	0.611	0.478	0.485	0.486	0.235	0.245	0.245
	0.5	0.444	0.439	0.438	0.345	0.345	0.344	0.169	0.171	0.170
		Inconsistent (IC): ρ_{XZ}=.20, ρ_{YZ}=.60								
		$\rho_{XY} = .20$			$\rho_{XY} = .50$			$\rho_{XY} = .80$		
n_r	π	FSI	BSI	BPI	FSI	BSI	BPI	FSI	BSI	BPI
20	0.1	3.405	2.417	1.828	3.046	2.170	1.713	1.627	1.296	1.159
	0.3	1.579	1.405	1.311	1.295	1.193	1.123	0.670	0.639	0.602
	0.5	1.117	1.073	1.042	0.904	0.892	0.869	0.476	0.488	0.472
60	0.1	2.476	1.882	1.610	2.017	1.614	1.428	0.992	0.842	0.781
	0.3	0.952	0.888	0.870	0.774	0.734	0.720	0.420	0.395	0.384
	0.5	0.667	0.642	0.636	0.535	0.521	0.516	0.285	0.280	0.276
100	0.1	1.967	1.601	1.448	1.636	1.365	1.256	0.821	0.699	0.654
	0.3	0.740	0.703	0.695	0.596	0.571	0.565	0.324	0.309	0.303
	0.5	0.522	0.508	0.505	0.419	0.411	0.408	0.223	0.220	0.218
		Consistent and more sizable (CM): ρ_{XZ}=.80, ρ_{YZ}=.60								
		$\rho_{XY} = .20$			$\rho_{XY} = .50$			$\rho_{XY} = .80$		
n_r	π	FSI	BSI	BPI	FSI	BSI	BPI	FSI	BSI	BPI
20	0.1	3.073	2.209	1.725	3.351	2.405	1.824	2.452	1.847	1.534
	0.3	1.641	1.466	1.347	1.485	1.390	1.284	0.874	0.909	0.854
	0.5	1.153	1.094	1.067	0.984	0.989	0.961	0.561	0.610	0.589
60	0.1	2.299	1.810	1.529	2.247	1.870	1.600	1.424	1.294	1.173
	0.3	0.955	0.924	0.903	0.857	0.851	0.828	0.491	0.513	0.499
	0.5	0.641	0.605	0.602	0.550	0.544	0.540	0.298	0.311	0.307
100	0.1	1.995	1.630	1.427	1.826	1.603	1.439	1.131	1.055	0.981
	0.3	0.725	0.714	0.707	0.646	0.654	0.644	0.362	0.382	0.374
	0.5	0.492	0.460	0.459	0.420	0.411	0.409	0.225	0.230	0.228

Note. ρ_{XY}, ρ_{XZ}, and ρ_{YZ} are the population correlations between variables X and Y, X and Z, and Y and Z, respectively. n_r is the restricted sample size. π is the selection ratio. FSI is the 95% formula standard interval. BSI is the 95% bootstrap standard interval. BPI is the 95% bootstrap percentile interval.