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Abstract

The objective of the present study is to identify cognitive functions that underlie achievements in mathematics, and to examine how deficiencies in these functions may predict mathematics learning disability (MLD). Identification of such functions will assist in finding the core deficits of MLD, establishing a more valid diagnosis of MLD, and identifying children at-risk for MLD. To achieve these objectives, the performance of 112 fifth graders was studied in seven quantitative and non-quantitative tests, and a standard mathematics test. Results indicated that the most significant functions for the prediction of MLD were computational automaticity and number-line representation. Deficiency in these functions, as well as in quantity comparison, working memory and reading, is significantly correlated with the frequency of failure in standard mathematics tests. The implications of the results for the diagnosing of MLD are discussed.

Mathematics learning disability (MLD) is a specific learning disability impairing arithmetic skills in otherwise normal children (Shalev, Manor & Gross-Tsur, 1997). Since diagnostic criteria of MLD remain unresolved, a common approach to identify children as having MLD is if their scores in a standardized achievement test in mathematics fall below a specific cutoff (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). Yet, since the broad nature of achievement tests is not always sensitive enough to the specific cognitive deficits which characterize children with MLD, the development of measures that are more sensitive to cognitive deficits underlying MLD remains a core goal of the research in this area (Geary, 2005; Gersten, Jordan, & Flojo, 2005; Mazzocco & Thompson, 2005). The core cognitive deficits assumed to underlie and characterize MLD include both specific (quantitative) and general (non-quantitative) functions, (Geary et al., 2007; Landerl, Bevan & Butterworth, 2004).

Characteristics of Mathematics Learning Disability

The most robust and consistent indicators of MLD are: (1) computational deficiencies (Geary, 1993, 2004; Geary, Hamson, & Hoard, 2000; Geary et al., 2007; Gersten et al., 2005; Russell & Ginsburg, 1984), and (2) number-sense deficiencies (Butterworth, 2005; Gersten & Chard, 1999; Landerl et al., 2004).

Though researchers agree on the centrality of these indicators, their source remains disputed. According to the "specific approach" the above functions represent deficits in the "number module" that deals with numerical representations (Gersten, Clarke, & Mazzocco, 2007; Landerl et al., 2004). Alternately, the general approach argues that these deficits stem from or reflect more general cognitive deficits, such as deficits in visuo-spatial representation, in memory (semantic long-term memory and working memory), or in basic language skills (Geary, 2004, 2005).

Computational Deficiencies

Children learning to compute employ a variety of procedural strategies (Geary, 2004; Lemaire & Siegler, 1995; Sherin & Fuson, 2005). There is a developmental transition from procedural-based strategies to memory-based strategies (Geary, 2004). As early as second grade, typical achieving students retrieve most problems from long-term memory. By the end of the 6th grade, computation automaticity reaches the level of that of adults (De Brauwer, Verguts, & Fias, 2006). Children with MLD (whether accompanied with reading disabilities or not) use the same strategies as younger, typically achieving children (e.g., "counting all" instead of "counting on", finger counting instead of verbal counting) for longer periods and in addition, they use them in a less efficient way, namely, they show longer response time and make more errors (Geary, 2004; Geary et al., 2007; Russell & Ginsburg, 1984). As early as kindergarten, computational deficiencies predict low mathematics achievements in the second and third grade (Mazzocco & Thompson, 2005). These deficiencies also delay the ability of children with MLD to understand the mathematical discourse and the more complex mathematical concepts presented in class (Gersten et al., 2007; Gersten et al., 2005).

Number Sense

Number sense is a conceptual structure that relies on many links among mathematical relationships, principles and procedures. These links serve as essential tools for helping students to think about mathematical problems and to develop higher order insights when working on mathematical problems (Gersten et al., 2005). Number sense is a necessary component of mathematical learning and is often compared to the role of phonological awareness in reading (Gersten & Chard, 1999). Deficiency in number sense often leads to ongoing problems in different domains of mathematics (Butterworth, 2005; Gersten & Chard, 1999; U.S. national mathematics advisory panel, 2008). In spite of its centrality, number sense is a vague concept (Gersten & Chard, 1999). In its most fundamental form, number sense entails an ability to immediately identify the numerical value associated with small quantities, a facility with basic counting skills and a proficiency in approximating the magnitudes of small numbers of objects. A more advanced type of number sense requires also a principle understanding of place value, of how whole numbers can be composed and decomposed, and of the meaning and the properties of the four basic arithmetic operations and how to apply them to solve problems (U.S. national mathematics advisory panel, 2008). Von Aster and Shalev (2007) suggested a four-step model of number development that is applicable to developmental MLD. The first step is cardinality, which represents a basic, inherent, quantitative, pre-verbal number sense. The cardinality concept is similar in a way to the terms "numerosity" (Landerl et al., 2004) and "magnitude" (Dehaene & Cohen, 1995). In the next two steps, linguistic (step 2) and Arabic (step 3) symbols are attached to this meaning of "number". The last step is the expanding number-line, a process that takes place during school years and is associated with the ordinary, sequential meaning of numbers.

Among the many tasks often used to assess number sense, counting, quantity comparison and number line representation are very common. Though the understanding of counting principles by children with MLD is delayed, it tends to reach maturation by second grade (Geary, 2004; Geary et al., 2007). Number line and quantity comparison represent two separate factors of number sense among young children. These two basic functions are precursors of other, more advanced components of number sense (Butterworth, 2005; Gersten et al., 2005).

Quantity Comparison

Children aged 11 months already differentiate between actions that lead to an increase or a decrease in a given amount. At kindergarten age, children can identify the larger of two sets, but only those children with a better number sense know how much larger it is, or if it is much larger or a little larger (Gersten et al., 2005).

Number-line Representation

The range of the number line representation increases with age and practice, though it is also affected by task features. There is a developmental transition from a logarithmic representation (in which the intervals between small numbers on the number line are bigger than between big numbers) to a linear representation (in which the intervals between all numbers are equal) (Siegler & Booth, 2004). Children with MLD use the logarithmic representation longer, and are less accurate than typically achieving children or children with mathematical difficulties without MLD (Geary et al., 2007). Number line representation was found to be related to and predictive of mathematics achievements among both kindergarten and school children (Mazzocco & Thompson, 2005; Siegler & Booth, 2004). Siegler and Booth claimed that constructing a mental number-line is essential for normal mathematical development and enables children to solve problems they were previously unable to cope with.

Visual-spatial Deficiencies

The visual basis of MLD is not quite clear. Visual function is considered to be related especially to multi-digit computation (Dehaene & Cohen, 1995; Geary, 2004). Nonetheless, several effects may indicate an activation of visual-spatial representations in a wider range of numerical processing (Ashcraft, 1995; Dehaene & Cohen, 1995; Dehaene, Piazza, Pinel, & Cohen, 2003; Geary, 2004; Von-Aster & Shalev, 2007). Hence, researchers emphasize the importance of normal visual-spatial perception for non-verbal magnitude representation (Dehaene et al., 2003). Accordingly, difficulties in visual perception are considered as one of the core-deficits underlying MLD (Geary, 1993, 2004; Jordan & Hanich, 2003, cited in Gersten et al., 2005). Research findings are mixed. Spatial ability was found to be related to mathematics achievements among college students, after general intellectual variance was removed (McGlaughlin, Knoop, & Holliday, 2005; Osmon, Smerz, Braun, & Plambeck, 2006). In contrast, Geary, Hamson, and Hoard (2000) found no spatial deficits among children with MLD, and Mazzocco and Thompson (2005) found that visual-spatial tests did not contribute to the prediction of MLD among school children.

Working Memory

Working memory is the ability to hold a mental representation of information in mind while simultaneously engaging in other mental processes. Working memory is composed of a central executive system and two slave systems: a language-based phonetic buffer and a visuo-spatial sketch pad. Various components of working memory are active when performing arithmetic tasks (Ashcraft, 1995): (1) Addition relies mainly on the central executive system. This reliance increases when the computational links are weak (as happens in MLD). (2) Counting relies mainly on the phonological loop, but also on the central executive system in order to separate the items that were counted from those that should be counted. Therefore, computation based on counting instead of on retrieval involves both the phonological loop and the central executive system. (3) There is no clear evidence regarding the function of the visual-spatial sketch pad in arithmetic actions. Ashcraft (1995) assumes that it has a secondary roll to that of the central executive system in multi-digit computation that requires a carry operation. Positive correlations were found in several studies between working memory and various arithmetic functions across ages (Bull & Scerif, 2001; Geary, 1993, 2004; Geary et al., 2007; Gersten et al., 2005). In contrast, Temple and Sherwood (2002) found no differences between children with MLD and typically achieving children in working memory, and no correlation between working memory and arithmetic ability. Geary (1993) hypothesized that the counting speed of children with MLD is slow, whereas the decay of the information is fast and hence, simultaneous activation of the problem and the answer is not created and thus not transferred to the phonological and semantic long-term memory. Furthermore, due to the inefficiency of their executive system, erroneous solutions are blocked to a lesser extent. Landerl, Bevan and Butterworth (2004) disagree with this assumption claiming that if this theory were correct, all dyslexic children would be expected to have number fact problems. Strong reading skills of the MLD group and intact fact retrieval skills of the reading learning disability group found in Jordan and Hanich's study (2003) support this claim.

Reading Ability

Co-morbidity of mathematics disability and reading disability is very common (Butterworth, 2005; Jordan, 2007; Jordan & Hanich, 2003; Shalev et al., 1997). Children with this double deficit (i.e., mathematics and reading) seem to be inferior in their learning ability to children with MLD only - their performance in mathematics is slower and more error-prone and their growth curve is more moderate. Both groups show a similar functional profile with respect to number processing, reflecting the most common and predictive numerical core deficits of MLD. Yet, some math difficulties (e.g., word problems) reflect language difficulties. Hence, reading difficulties appear to aggravate rather than cause math difficulties. Compensatory mechanisms associated with reading and language are less available to children with MLD accompanied with reading disability, than to those with MLD only (Jordan, 2007; Jordan & Hanich, 2003; Landerl et al., 2003; McGlaughlin et al., 2005; Shalev et al., 1997).

The Present Study

Mathematics learning requires diverse cognitive functions, even at the primary school level (Jordan, 2007; Mazzocco & Thompson, 2005). The objective of the present study is to identify cognitive functions that not only underlie and predict achievements in mathematics, but that are a pre-requisite to these achievements. Identification of such functions may assist in: finding the core deficits of MLD, establishing a more valid diagnosis of MLD, identifying children at-risk for MLD and differentiating between children with MLD and children who have mathematical difficulties for other reasons.

To achieve the above objectives, the performance of 112 fifth graders was studied in seven quantitative and non-quantitative tests, and a standard mathematics test. To examine the degree to which each of the above functions may be a "pre-requisite" for normal mathematical functioning, the frequency of failure in a standard mathematics test as a function of deficiency in the above cognitive tests was analyzed.

Method

Participants

The research sample consisted of 112 fifth-grade students. Fifty-three percent of the participants were boys and 47% were girls. Twenty percent of the participants belonged to a low socioeconomic status (SES), 61% to a medium SES, and 19% to a high SES. In order to ensure diversity of math instruction methods, participants were randomly selected from 19 different schools, located in different communities. To allow for diversity in learning abilities and difficulties, 19 (17%) participants, previously diagnosed as having LD or at-risk for LD, were included in the study. The percentage of children having LD or at-risk for LD is higher in the present study than the common prevalence of mathematical LD, which ranges from 3-6.5% (Shalev, 2004).

Tools

The study materials consisted of a background questionnaire and eight tests, listed below, that evaluate various aspects of learning.

*Computational automaticity***.** The test assesses the ability to retrieve basic numerical facts. The test consists of four practice trials followed by 64 true/false tasks. In each task, participants are presented with a correct or incorrect simple arithmetic equation (e.g., 4 + 2 = 6; 4 + 2 = 8) and are asked to indicate its correctness by clicking the right arrow key for a correct equation and the left arrow key for an incorrect equation. All the equations are single-digit operands. There are 16 equations for each of the four arithmetical operations. Measures of accuracy and response time are obtained. The test is based on a standard paradigm but the equations were developed by the current authors.

Quantity comparison. The test assesses the ability to compare quantities. The test consists of two practice trials, followed by 40 tasks. In each task, participants are presented with two clusters of dots (one on the right side of the computer screen and the other on the left side). They are asked to indicate which cluster contains a larger quantity of dots by clicking the arrow keys. Measures of accuracy and reaction time are obtained. The task is based on Barth, Kanwisher and Spelke's (2003) paradigm

Number line. The test assesses number line representation through the ability to match a number to its position on a number line. The test consists of six practice tasks, followed by 40 tasks. In each task, a number line with two anchor points indicating a segment magnitude (e.g., a line with the points 1 and 10 marked off) is presented to the participants. Two additional points are marked off on this segment, with the same value attributed to both of them (e.g., two points with the value 5). One of these two points corresponds to the attributed numerical magnitude relative to the line segment, while the other does not. Participants are required to click the arrow key (right or left) corresponding to the correct target (i.e., the point whose position corresponds with the attributed numerical magnitude within the line segment). Measures of accuracy and reaction time are obtained. The test is based on a paradigm developed by Ben-Simon (2008).

Visual perception. The test assesses parallel spatial frequency discrimination between two horizontal sinusoidal gratings. On each trial one stimulus contains the reference frequency, which remains constant through the test. The other stimulus contains the test frequency, which is randomly selected to be either higher or lower than the reference frequency. The initial test frequency is differed by 75% from the reference frequency and it varies adaptively between trials in a tow down / 1 up staircase manner, which converges on the value of 71% correct. Initial step size is 10% and it is halved every three reversals (to a minimum of 1%). Participants are requested to indicate which grid is denser by clicking on the arrow keys: ↑ for the upper grid and

↓ for the lower grid. The test is terminated after 15 reversals but no more than 80 trials. Discrimination threshold—Just Noticeable Difference (JND in % of reference frequency)—is determined as the average of the last ten reversals. The test is based on Ben-Yehuda and Ahissar's paradigm (2004).

*Working memory***.** The test assesses temporal spatial frequency discrimination. The test paradigm is identical to that of the Visual Perception Test (parallel spatial frequency discrimination) except for the fact that in this test, the stimuli appear temporally (i.e., one after the other). Hence, the ability of the working memory is assessed.

Pseudo words. The test is a standard test used for the assessment of decoding (Shany, Lachman, Shalem, Bahat & Zeiger., 2006). The test consists of four practice trials followed by 33 vocalized non-words (with diacritic signs). Participants are asked to read each non-word aloud. Measures of accuracy and response time are obtained. The internal consistency reliability coefficient reported by the developers is 0.90 for accuracy.

Text reading. Accuracy and fluency of text reading were examined by a standard test in Hebrew (Shany et al., 2006). The text used is a 196-word informational text. The reliability coefficients reported by the developers are 0.89 for accuracy and 0.80 for fluency.

Mathematics achievement. The test is a standard national exam that was administered nation wide in 2005 to approximately 35,000 students. The test consists of 33 questions, in both multiple-choice and open-ended format, and covers various topics from the national curriculum for 5th graders. The final score is given on a scale of 0-45, which is later transformed to a scale of 0-100.

Background questionnaire. The questionnaire consists of nine questions covering various background variables such as: gender, age, school type and location, parents' education and previous diagnosis of learning disability (if applicable).

Procedure

All tests were administered individually by four trained examiners with M.Ed. degrees, during a period of three months. The tests were administered in a fixed order. Average testing time was about 1.5 hours. Participation in the study was contingent on parental consent and was remunerated.

Results

The performance of 112 fifth graders was studied in seven quantitative and non-quantitative tests, and a standard mathematics test described above. Pearson correlations, factor analysis and ANOVA (analysis of variance) procedures were carried out to examine the relationships between cognitive functions and achievements in mathematics. To examine the degree to which each of the cognitive functions may be a pre-requisite for mastery in mathematics, the frequency of failing the standard mathematics test as a function of deficiency in the above cognitive tests was analyzed.

Scores Distribution

Table 1 shows the mean, standard deviation and reliability coefficient obtained for the performance measures of all tests. The mean score obtained on the Mathematics Achievement Test was lower by approximately 0.5 *SD* than the national mean. This result can be attributed in part to the inclusion of a relatively large number of students with learning difficulties, as mentioned above. The mean accuracy and pace of reading found for both Text Reading and Pseudo Words tests fits the norms of fifth graders. The mean thresholds obtained in the Visual Perception (11.9%) and the Working Memory (19.8%) tests were only slightly different from those of adult students (16.0% and 12.2%, respectively). Relatively high scores were obtained for accuracy in the Quantity Comparison test (91.5%) and for accuracy in the Text Reading test. These results indicate a possible ceiling effect. High reliability coefficients (.82 - .96) were found for all performance measures except for quantity comparison accuracy (.43). This fairly low reliability coefficient can probably be attributed to the ceiling effect of the scores' distribution.

Table 1

Mean, Standard Deviation (SD) and Reliability Coefficient for all Performance Measures

Test	**Performance measure**[1]	**Study results**			**National norms**	
		Mean	**SD**	**Reliability**	**Mean**	**SD**
Mathematics achievements	Total	68.4	20.0		75.5	15.7
Computational automaticity	Accuracy	83.0	13.6	.90		
	RT	2428	640	.96		
Quantity comparison	Accuracy	91.5	4.5	.43		
	RT	1312	368	.88		
Number line	Accuracy	81.6	13.0	.82		
	RT	3095	1070	.91		
Visual perception	Threshold	19.8	22.0		16.0[2]	3.3[3]
Working memory	Threshold	11.9	7.0		12.2[2]	2.6[3]
Pseudo-words	Accuracy	60.9	21.1	.90[4]	64.2	22.0
	Pace	19.7	7.1	.90[4]	23.1	10.4
Text reading	Accuracy	94.4	5.9	.89[4]	94.2	3.1
	Pace	92.9	23.7	.80[4]	92.5	33.6

[1] Performance measures: RT - response time (average number of ms / item); Accuracy - percentage of correct responses; Reading pace - number of words per minute.

[2] Mean threshold obtained for normative sample of adult students (age 18-26 years)

[3] Reliability - standard deviation between thresholds obtained in the last 15 steps

[4] Reliability coefficients obtained for normative sample of fifth grade students

Relationships Between Cognitive Functions and Achievements in Mathematics

Table 2 shows the correlations (Pearson's product moment) between the cognitive functions and achievements in mathematics. Moderate to high significant correlations ($p < .01$) were found for accuracy in computational automaticity (.67), accuracy in number line representation (.54), accuracy in text reading (.48), accuracy in pseudo word reading (.36), and pace of text reading

(.29). Significant negative correlations ($p < .05$) were found for visual perception (.21) and response time of computational automaticity (.19). Non significant correlations were found for both accuracy and reaction time (RT) of quantity comparisons, for RT of number line representation and for pseudo words reading pace.

In order to further explore the relationships between various cognitive functions and achievements in mathematics, factor analysis—oblique rotation—was carried out. Table 3 shows the results. In light of the high correlations found between the reading functions and achievements, two separate scores were extracted from the Mathematics Achievement Test: (1) a quantitative-computational score, which included results from 20 number-problems; and (2) a quantitative-verbal score, which included results from word-problems and tasks that required conceptual knowledge with no computation (25 items). The correlation between the scores was .37.

Table 2

Correlations Between Cognitive Functions and Achievements in Mathematics

Test	**Performance measures**	**Correlation**
Computational automaticity	Accuracy	.67**
	RT	-.19*
Quantity comparison	Accuracy	.13
	RT	.02
Number-line	Accuracy	.54**
	RT	.01
Visual perception	Threshold	-.21*
Working memory	Threshold	-.34**
Pseudo words	Accuracy	.36**
	Pace	.05
Text reading	Accuracy	.48**
	Pace	.29**

** $p < .01$
* $p < .05$

Table 3

Factors Analysis of the Tests

	Factor 1	**Factor 2**	**Factor 3**	**Factor 4**	**Factor 5**
Quantitative-computational component in mathematical achievements	**.98**	-.01	-.01	0	0
Quantitative-verbal component in mathematical achievements	**1.00**	0	0	0	-.01
Computational automaticity – accuracy	**.99**	-.01	0	0	0
Computational automaticity – RT	-.01	0	**.89**	0	-.06
Quantity comparison – accuracy	0	**-.61**	.06	.04	-.02
Quantity comparison – RT	0	**-.53**	.07	-.06	-.02
Number-line - accuracy	**.79**	0	.06	0	-.01
Number-line - RT	0	0	**1.00**	0	0
Visual perception	-.01	0	0	0	**1.00**
Working memory	-.07	.06	0	0	**.58**
Pseudo words – accuracy	-.03	**1.00**	0	0	0
Pseudo words - pace	0	0	0	**1.00**	0
Text reading - accuracy	-.15	**.54**	0	-.05	0
Text reading - pace	.02	-.01	0	**.71**	-.03

Note:
Factor 1 = accuracy in quantitative functions,
Factor 2 = phonological decoding,
Factor 3 = processing speed of quantitative information,
Factor 4 = reading pace,

Factor 5 = visual perception.

The factor analysis yielded five factors (see Table 3): (1) accuracy in quantitative functions, (2) phonological decoding, (3) processing speed of quantitative information, (4) reading pace, and (5) visual perception.

A stepwise regression was used to examine the extent to which the performance in the various cognitive functions could predict achievements in mathematics. Three variables had a significant contribution to the prediction of achievements in mathematics ($F = 13.8$, $p = .011$): accuracy of computational automaticity, accuracy in number line representation and computational automaticity RT. The three variables explained 57% of the variance in achievements. Results are shown in Table 4.

Table 4

Multiple Linear Analysis of Mathematical Achievements

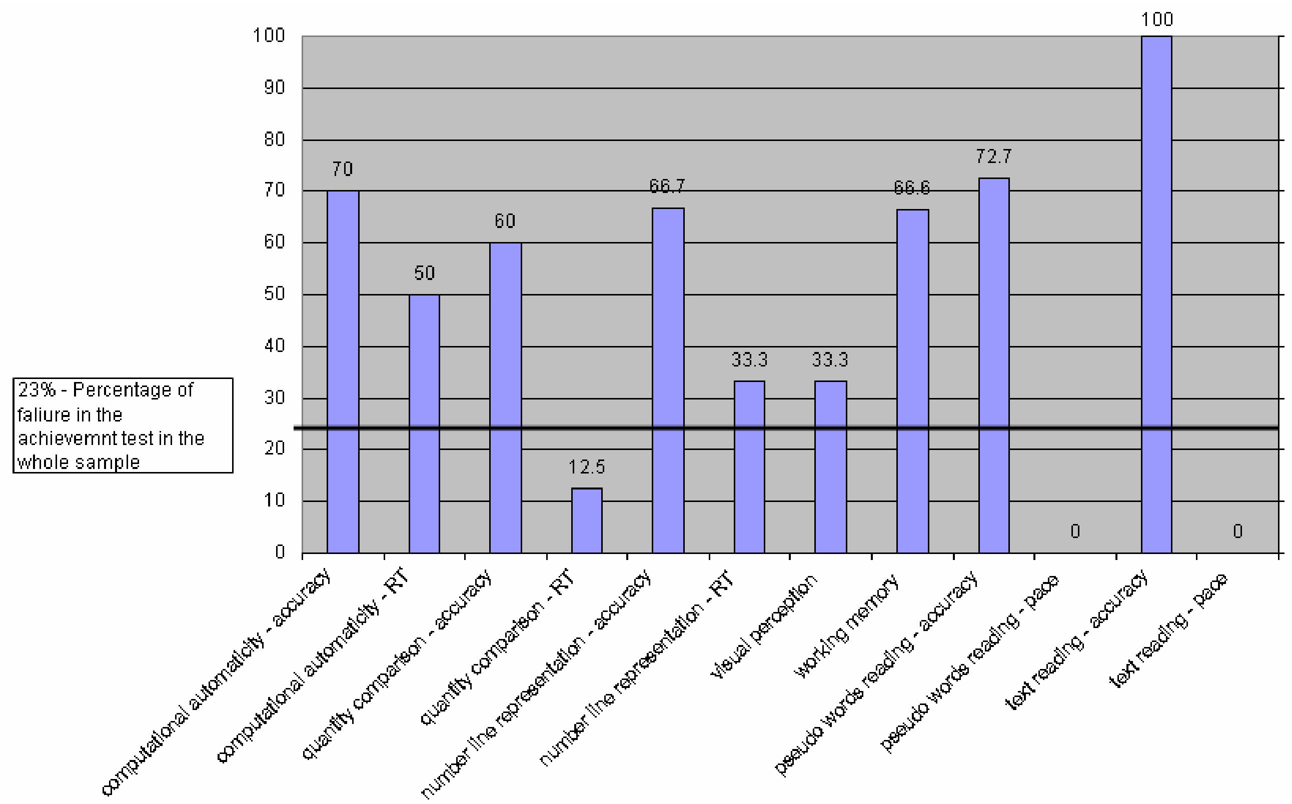
Tests	R^2	**Marginal addition to** R^2	F	P
Computational automaticity – accuracy	.44	.44	84.2	< .001
Number line – accuracy	.51	.07	14.1	< .001
Computational automaticity - pace	.57	.06	13.8	.011
Text reading – accuracy	.60	.03	6.7	.146
Quantity comparison – accuracy	.61	.01	2.5	.117

Identification of Pre-requisite Functions for Normal Mathematical Achievements

To examine the degree to which each of the cognitive functions may be a "pre-requisite" for mastery in mathematics, the frequency of failures in the standard mathematics test as a function of deficiency in the above cognitive tests was analyzed.

A cutoff score of 1.5 standard deviations below average was defined as differentiating between deficient and non-deficient performance on the cognitive tests. The percentage of students with deficient performance on these tests ranged from 2.6% to 10.7%. The criterion for determining failure in the Mathematics Achievements Test was a score of 55 (the common standard in the education system). Twenty-three percent (26 out of 112) of the students in the study failed the Mathematics Achievements Test based on the above criterion. Figure 1 shows the percentage of students who failed the Mathematics Achievement Test out of those who were found to have "deficient" performance in each of the cognitive tests. The higher the failure percentage of a function, the more likely it is that the given function serves as a pre-requisite for proficiency in mathematics.

Figure 1. Percentage of participants who failed the Standard Mathematics Achievement Test among those who were deficient in each cognitive test.



Results show that all of the students who were deficient in accuracy in text reading failed the Mathematics Achievement Test. High failing rates in the Mathematics Achievement Test were also found among students who had deficient performance in the following cognitive functions: accuracy in pseudo words reading (72.7%), accuracy in computational automaticity (70%), accuracy in number line representation (66.7%), working memory (66.6%) and accuracy in quantity comparisons (60%).

Discussion

Defining MLD is challenged by the complexities associated with school mathematics and the variety of cognitive skills required for achieving mathematical proficiency. A wide range of cognitive abilities support successful mathematics, and efforts to define MLD should find which deficits of these functions are essential features of MLD. Achievement scores should be used only as a starting point in the diagnostic process (Geary et al., 2007; Mazzocco, 2007). Hence, a core goal of research on MLD is to find tools that are more sensitive than achievement tests to the specific deficits of children with MLD (Geary, 2005).

The objective of the present study, following the above goal, was to identify cognitive functions that not only underlie and predict achievements in mathematics, but also comprise a pre-requisite to these achievements. Although correlations between a wide range of cognitive functions and achievement were extensively reported in previous studies, the degree to which proficiency in mathematics is conditioned on the normal performance of these functions was not systematically studied. To achieve this objective, the performance of 112 fifth graders was studied in three quantitative and four non-quantitative tests, as well as a standard mathematics test.

Given the dispute regarding the specificity of mathematical dysfunctions, the results of the study are reported separately for the quantitative (specific) and non-quantitative (general) functions.

Specific (quantitative) function. Three quantitative functions were examined in the study: computational automaticity, quantitative comparisons and number-line representation. All three functions were identified in previous studies as highly associated with achievements in mathematics and were found deficient in MLD. While computational automaticity is one of the main components of computational skills, quantity comparisons and number-line representation are regarded as components of number sense. Two out of the above three functions were found to be highly correlated with achievements: computational automaticity (r = .67) and number-line representation (r = .54). In both functions the accuracy performance measure was a far better indicator of low achievement as opposed to the RT measure.

Compatible with the correlation results, the "pre-requisite" analysis showed that the frequency of students who failed the standard mathematics test increased from 23% (in the full sample) to 70% among students with deficiency in computational automaticity, and to 66.7% among students with deficiency in number-line representation. These findings are concordant with previous findings indicating that computational deficiencies (Geary, 1993, 2004; Geary et al, 2000; Geary et al., 2007; Gersten et al, 2005) and number-sense deficiencies (Butterworth, 2005; Gersten & Chard, 1999; Landerl et al., 2004) are the most robust and consistent indicators of MLD.

Contrary to previous findings, quantitative comparison ability was not correlated with achievements (.13 for accuracy). It should be noted that the fairly low correlation observed for the quantitative comparison may be attributed to a ceiling effect, namely most students had mastered this skill at fifth grade. Yet, deficiency in quantity comparison notably increased the failing rate on the standard test (from 23% to 60%), thus emphasizing its importance in the acquisition of mathematic ability, as argued by Butterworth (2005) and Dehaene et al. (2003).

Non-quantitative functions. Four non-quantitative functions were studied with relation to achievements in mathematics: Visual perception, working memory, pseudo-word reading and text reading. Of the four functions, accuracy in text reading (.48) was found to be the most related to achievements in mathematics, followed by accuracy in pseudo-words reading (.36), text reading pace (.29), working memory (-.34) and visual perception (-.21). The correlations between the non-quantitative functions and achievements were generally lower than those obtained for the quantitative functions. Nonetheless, the pre-requisite analysis reveals a somewhat different picture as to the relevance of three out of these four functions for proficiency in mathematics. While only 23% of the students in the whole sample failed the standard test, practically all of the students (100%) with deficiency in text reading failed this test. High failing rates were also found among students who were deficient in pseudo words reading (72.7%) and in working memory (66.6%). As in the case of the quantitative functions, reading pace of either text or pseudo-words had no bearing on achievements in mathematics.

The relationship between reading ability and mathematics achievements is well documented in the research literature, although its nature is not quite elaborated on. The various explanations offered for this relationship include the significant role of reading in the understanding of instructions and word-problems (Jordan, 2007), and the common G (general intelligence) factor that underlies higher cognitive functions (Osmon et al., 2006). Both visual perception and working memory were often mentioned in theoretical models as important mechanisms related to mathematical functioning and potential sources of MLD (Geary et al., 2007). The current study suggests that if they indeed play a role in mathematics functions, it is mostly one as a necessary condition, namely, mastery at a minimal basic level is required as a pre-condition for proficiency in mathematics.

Implications for diagnosing MLD. The diagnosis of MLD is often applied in two different contexts: (1) early identification of children at risk; and (2) identification of the sources underlying difficulties in mathematics for the development of an intervention plan. The first context calls for an efficient screening procedure while the second requires a more in-depth assessment.

In agreement with Mazzocco and Thompson (2005), results of the current study indicate that only quantitative functions contribute significantly to the explained variance of mathematical achievements; computational automaticity and number-line representation explain 57% of the variance of the achievement scores. Therefore, it may be argued that the assessment of these two functions might suffice for screening purposes.

Concerning identification of the sources underlying difficulties in mathematics for the development of an intervention plan, the assessment of a wider variety of quantitative and non-quantitative cognitive functions is required. The results of the current study suggest that the functions that should be included in such a procedure are those which were found to be highly correlated with achievements (i.e., computational automaticity and number line representation), as well as functions which were identified as pre-requisites of mathematical proficiency, namely, quantity comparison, reading and working memory.

The current study focused on a rather limited number of both quantitative and non-quantitative functions. Further research is required to examine the role of other functions at various grade levels.

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