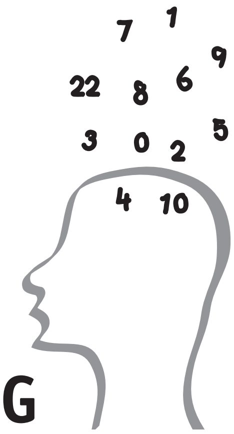


QUANTITATIVE REASONING

The quantitative reasoning section tests your ability to use numbers and mathematical concepts to solve quantitative problems, and your ability to analyze data presented in different ways, such as table or graph form. This section requires only basic knowledge of mathematics (the material studied up to 9th-10th grades in most Israeli high schools).

- Several types of questions make up the quantitative reasoning section: Questions and Problems, Graph or Table Comprehension, and Quantitative Comparisons (examples of each type appear later on in the *Guide*).

  Questions and Problems: These are multiple-choice questions (a question followed by four possible responses). They cover a variety of subjects, such as distance problems, work problems, combinatorial analysis, probability, equations, geometry and so on. Some are non-verbal questions in which the problem is presented numerically; others are verbal questions, which require that the problem be translated into mathematical terms; other questions deal with characteristics of geometrical figures, such as area, angles and so on.

  Graph or Table Comprehension: These are multiple-choice questions which relate to information appearing in a graph or a table. A table presents numerical data arranged in columns and rows. A graph presents data in graphic form, such as a curve or a bar chart. There are two main types of questions:

  - Questions involving the reading of data, in which you are asked to find information appearing in the graph or table.
  - Questions in which you are asked to make various inferences based on the data appearing in the graph or table.

  Quantitative Comparisons: These questions cover a variety of topics. They consist of pairs of quantities; in some cases additional information is provided. In each question, you are asked to decide, on the basis of the quantities and the additional information (if provided), whether one of the quantities is larger than the other, whether the two quantities are equal, or whether there is not enough information to determine the relationship between the two quantities.

- In general, all questions of a given type are arranged in ascending order of difficulty. In other words, the easier questions, requiring less time to solve, appear first, with the questions becoming progressively more difficult and requiring more time to solve.
- The figures accompanying some of the questions are not necessarily drawn to scale. Do not rely solely on the figure's appearance to deduce line length, angle measure and so forth, unless these are specified in the figure (or in the question itself). But if a line in a figure appears to be straight, you may assume that it is, in fact, a straight line.
- A page of Symbols and Formulas appears at the beginning of each quantitative reasoning section. This page contains instructions, general comments and formulas, which you may refer to during the test. The page of Symbols and Formulas also appears on p. 35 of the Guide and in the quantitative reasoning sections of the practice test. You should familiarize yourself with its contents prior to taking the test.

Pages 36-58 contain a review of basic mathematical concepts, covering much of the material upon which the questions in the quantitative reasoning sections are based. The actual test may, however, contain some questions based on mathematical concepts and theorems that do not appear on these pages.

Pages 59-79 contain examples of different types of questions, each followed by a detailed explanation.

Quantitative Reasoning

This section contains 25 questions.
The time allotted is 25 minutes.

This section consists of questions and problems involving quantitative reasoning. Each question is followed by four possible responses. Choose the correct answer and mark its number in the appropriate place on the answer sheet.

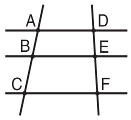
Note: The words appearing against a gray background are translated into several languages at the bottom of the page.

General Comments about the Quantitative Reasoning Section

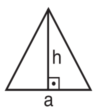
* The figures accompanying some of the questions are provided to help in answering the questions, but are not necessarily drawn to scale. Therefore, do not rely on the figures alone to deduce line length, angle measure, and so forth.
* If a line in a figure appears to be straight, you may assume that it is in fact a straight line.
* When a geometric term (side, radius, area, volume, etc.) appears in a question, it refers to a term whose value is greater than 0, unless stated otherwise.
* When $\sqrt{a}$ $(a > 0)$ appears in a question, it refers to the positive root of a.

Symbols and Formulas

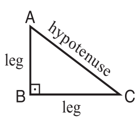
1. **The symbol** $\llcorner$ represents a 90° (right) angle.
   The symbol $\sphericalangle ABC$ represents the angle formed by line segments AB and BC.
   $a \parallel b$ means a is parallel to b.
   $a \perp b$ means a is perpendicular to b.
2. **Zero** is neither a positive nor a negative number.
   Zero is an even number.
   One is not a prime number.
3. **Percentages:** $a\%$ of x is equal to $\frac{a}{100} \cdot x$
4. **Exponents:** For every a that does not equal 0, and for any two integers n and m -
   a. $a^{-n} = \frac{1}{a^n}$ b. $a^{m+n} = a^m \cdot a^n$
   c. $a^{\frac{n}{m}} = (\sqrt[m]{a})^n$ $(0 < a, 0 < m)$ d. $a^{n \cdot m} = (a^n)^m$
5. **Contracted Multiplication Formulas:**
   $(a \pm b)^2 = a^2 \pm 2ab + b^2$
   $(a + b)(a - b) = a^2 - b^2$
6. **Distance Problems:** $\frac{\text{distance}}{\text{time}}$ = speed (rate)
7. **Work Problems:** $\frac{\text{amount of work}}{\text{time}}$ = output (rate)
8. **Proportions:** If $AD \parallel BE \parallel CF$ then $\frac{AB}{DE} = \frac{BC}{EF}$ and $\frac{AB}{AC} = \frac{DE}{DF}$



9. **Triangles:**
   a. **The area of a triangle** with base of length a and altitude to the base of length h is $\frac{a \cdot h}{2}$

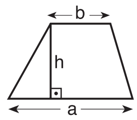


   b. **Pythagorean Theorem:**
   In any right triangle ABC, as in the figure, the following always holds true: $AC^2 = AB^2 + BC^2$

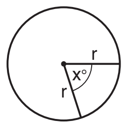


   c. In any right triangle whose angles measure 30°, 60° and 90°, the length of the leg opposite the 30° angle is equal to half the length of the hypotenuse.

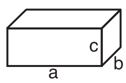
10. **The area of a rectangle** of length a and width b is $a \cdot b$
11. **The area of a trapezoid** with one base a, the other base b, and altitude h is $\frac{(a + b) \cdot h}{2}$



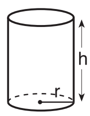
12. **The sum of the internal angles of a polygon** with n sides is $(180n - 360)$ degrees.
    In a regular polygon with n sides,
    each internal angle measures
    $\left(180 - \frac{360}{n}\right) = \left(\frac{180n - 360}{n}\right)$ degrees.
13. **Circle:**
    a. **The area** of a circle with radius r is πr^2 $(\pi = 3.14...)$
    b. **The circumference** of a circle with radius r is $2\pi r$
    c. **The area of a sector of a circle** with a central angle of $x°$ is $\pi r^2 \cdot \frac{x}{360}$



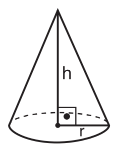
14. **Box (Rectangular Solid), Cube:**
    a. **The volume** of a box of length a, width b and height c is $a \cdot b \cdot c$
    b. **The surface area** of the box is $2ab + 2bc + 2ac$
    c. In a **cube**, $a = b = c$



15. **Cylinder:**
    a. **The lateral surface area** of a cylinder with base radius r and height h is $2\pi r \cdot h$
    b. **The surface area** of the cylinder is $2\pi r^2 + 2\pi r \cdot h = 2\pi r(r + h)$
    c. **The volume** of the cylinder is $\pi r^2 \cdot h$



16. **The volume of a cone** with base radius r and height h is $\frac{\pi r^2 \cdot h}{3}$



REVIEW OF BASIC MATHEMATICAL CONCEPTS

SYMBOLS

Below is a list of commonly used symbols that may appear on the test.

Symbol	Meaning
$a \parallel b$	straight lines a and b are parallel
$a \perp b$	straight line a is perpendicular to straight line b
⦜	90º angle (right angle)
$\sphericalangle ABC$	the angle formed by sides AB and BC
$x = y$	x equals y
$x \neq y$	x does not equal y
$x < y$	x is smaller than y
$x \leq y$	x is smaller than or equal to y
$0 < x , y$	both x and y are greater than 0
$x = \pm a$	x may be equal to a or to (-a)
$\lvert x \rvert$	the absolute value of x: if $0 < x$, then $x = \lvert x \rvert$ if $x < 0$, then $-x = \lvert x \rvert$ $0 = \lvert 0 \rvert$
$x : y$	the ratio of x to y

TYPES OF NUMBERS

Integer:	An integer is a number composed of whole units. An integer may be positive or negative; 0 is also an integer. For example: ... , -4 , -3 , -2 , -1 , 0 , 1 , 2 , 3 , 4 , ...
Non-integer:	A number that cannot be expressed in whole units. For example: 1.37 , $2\frac{1}{2}$
Consecutive numbers:	Integers that follow in sequence in differences of 1. For example, 4 and 5 are consecutive numbers; (-3) and (-2) are also consecutive numbers. In general, if n is an integer, then n and (n + 1) are consecutive numbers.
Even number:	An integer which, when divided by 2, produces an integer (in other words, it is evenly divisible by 2). Note that based on this definition, 0 is an even number. In general, if n is an integer, then 2n is an even number.
Odd number:	An integer which, when divided by 2, produces a non-integer (in other words, it is not evenly divisible by 2). In general, if n is an integer, then 2n+1 is an odd number.

Prime number:	An integer that is evenly divisible by only two numbers – itself and the number 1. For example, 13 is a prime number because it is evenly divisible only by 1 and 13. Note that 1 is not a prime number.
Reciprocal numbers	A pair of numbers which, when multiplied, equal 1. Examples: For $a \neq 0$, $b \neq 0$ a and $\frac{1}{a}$ are reciprocal numbers; $a \cdot \frac{1}{a} = 1$ $\frac{a}{b}$ and $\frac{b}{a}$ are reciprocal numbers; $\frac{a}{b} \cdot \frac{b}{a} = 1$
Opposite numbers	A pair of numbers whose sum equals zero. For example, a and $(-a)$ are opposite numbers. In other words, $(-a)$ is the opposite number of a $(a + (-a) = 0)$.

ARITHMETICAL OPERATIONS WITH EVEN AND ODD NUMBERS

even	+	even	=	even
odd	+	odd	=	even
odd	+	even	=	odd
even	–	even	=	even
odd	–	odd	=	even
odd	–	even	=	odd
even	–	odd	=	odd
even	×	even	=	even
odd	×	odd	=	odd
odd	×	even	=	even

There are no similar rules for division. For example, the quotient of two even numbers may be odd $\left(\frac{6}{2} = 3\right)$, even $\left(\frac{4}{2} = 2\right)$, or a non-integer $\left(\frac{6}{4} = 1\frac{1}{2}\right)$.

DIVISORS AND MULTIPLES

Factor (Divisor):

The factor of a positive integer is any positive integer that divides it evenly. For example, the numbers 1, 2, 3, 4, 6, 8, 12 and 24 are factors of 24.

Common Factor (Common Divisor):

A common factor of x and y is a number that is a factor of x and also a factor of y.
For example, 3 is a common factor of both 24 and 30.

Prime Factor (Prime Divisor):

A prime factor is a number that is a factor (divisor) of some other number and is itself a prime number. For example, 2 and 3 are the prime factors of 24. Any positive integer (greater than 1) can be written as the product of prime factors.
For example, $24 = 3 \cdot 2 \cdot 2 \cdot 2 = 3 \cdot 2^3$

Multiple:

A multiple of an integer x is any integer that is evenly divisible by x. For example, 16, 32 and 88 are multiples of 8.

MATHEMATICAL OPERATIONS WITH FRACTIONS

Reduction:

When the numerator and the denominator of a fraction have a common factor, each can be divided by the same number, and the resulting fraction is equivalent to the original fraction with a smaller denominator. For example, if we divide the numerator and the denominator of $\frac{16}{12}$ by 4, the result is $\frac{4}{3}$ $\left(\frac{16}{12} = \frac{4}{3}\right)$.

Multiplication:

To multiply two fractions, multiply the numerators by each other and the denominators by each other.

For example: $\frac{2}{3} \cdot \frac{5}{7} = \frac{2 \cdot 5}{3 \cdot 7} = \frac{10}{21}$

Or in general: $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$

Division:

To divide a number (integer or fraction) by a fraction, multiply the number by the reciprocal of the divisor.
(The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$)

For example: $\frac{2}{5} : \frac{3}{8} = \frac{2}{5} \cdot \frac{8}{3} = \frac{2 \cdot 8}{5 \cdot 3} = \frac{16}{15}$

Or in general: $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} : \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}$

To multiply or divide an integer by a fraction, the integer can be regarded as a fraction whose denominator is 1, for example: $2 = \frac{2}{1}$

Addition and Subtraction:

To add or subtract fractions, they must be converted into fractions that have a common denominator. A **common denominator** is a number that is evenly divisible by the denominator of each of the fractions. After finding a suitable common denominator, each of the fractions must be converted into a fraction that has this common denominator. To do so, multiply the numerator and denominator of each of the fractions by the same integer, so that the number obtained in the denominator will be the number that was chosen to be the common denominator. Since the numerator and denominator are multiplied by the same number, the fraction has actually been multiplied by 1, and its value has not changed. After converting the fractions so that they have a common denominator, add or subtract the new numerators that were obtained, and reduce to lowest terms where possible.

For example, to solve the problem: $\frac{3}{4}+\frac{1}{6}+\frac{5}{8}$

24 is a possible common denominator, since it is evenly divisible by the denominators of each of the fractions: $24 : 4 = 6$, $24 : 6 = 4$, $24 : 8 = 3$

We will now convert each of the fractions into fractions with this common denominator:

To convert $\frac{3}{4}$ into a fraction whose denominator is 24, multiply the numerator and the denominator by 6: $\frac{3 \cdot 6}{4 \cdot 6} = \frac{18}{24}$

To convert $\frac{1}{6}$ into a fraction whose denominator is 24, multiply the numerator and the denominator by 4: $\frac{1 \cdot 4}{6 \cdot 4} = \frac{4}{24}$

To convert $\frac{5}{8}$ into a fraction whose denominator is 24, multiply the numerator and the denominator by 3: $\frac{5 \cdot 3}{8 \cdot 3} = \frac{15}{24}$

Next, add up only the numerators: $\frac{18}{24}+\frac{4}{24}+\frac{15}{24} = \frac{18+4+15}{24} = \frac{37}{24}$

Percentages

Percentages are a specific case of fractions: a% of x is $\frac{a}{100} \cdot x$. In these questions, convert the percentages to hundredths, and solve as in normal fraction problems.

Example 1: What is 60 percent of 80? (or: What is 60% of 80?)
Instead of 60 percent, substitute 60 hundredths, express the question in mathematical terms, and solve it as you would a normal multiplication of fractions:

$\frac{60}{100} \cdot 80 = \frac{60 \cdot 80}{100} = 6 \cdot 8 = 48$

Thus, 60% of 80 is 48.

Example 2: Joe had to pay 15 shekels tax on the 50 shekels that he earned. What percent is the tax?

The question is actually "What percent of 50 is 15?"
Convert the question into a mathematical expression: $\frac{x}{100} \cdot 50 = 15$

and solve the equation for x: $\frac{x}{2} = 15$

Thus, $x = 30$. In other words, 15 is 30% of 50, which is the percentage of the tax.

For questions that involve a change expressed as a percentage, convert the question into one of the two general formats presented in examples 1 and 2 (what is x percent of y, or what percent of y is x), and solve as a fraction problem.

Example 3: The price of an item that cost 80 shekels was raised by 25%. What is the new price?
Questions dealing with change expressed as a percentage generally involve a percent of the original price unless otherwise specified. Since 25% was added to the old price, the new price is 125% of the old price (100% + 25%). Therefore, you must calculate what 125% of 80 is (as in example 1).
Substitute hundredths for percent and solve $\frac{125}{100} \cdot 80 = 100$. The new price is 100 shekels.

Example 4: The change in the price of a certain item is given, and you are asked to calculate the change as a percentage.
For example, the price of an item dropped from 15 shekels to 12 shekels. By what percentage did the price drop?
The difference in the price is 3 shekels out of 15 shekels. You have to calculate what percent of 15 is 3 (similar to example 2).
Convert the question into a mathematical expression: $\frac{x}{100} \cdot 15 = 3$, and solve the equation for x: $x = \frac{3 \cdot 100}{15} = 20$
Thus, the price dropped by 20%.

Ratio

The ratio of x to y is written as $x : y$.

For example, the ratio between the number of pairs of socks and the number of shirts that Eli has is 3 : 2. In other words, for every 3 pairs of socks, Eli has 2 shirts. Stating it differently, the number of socks that Eli has is $\frac{3}{2}$ greater than the number of shirts that he has.

Arithmetic Mean

The **arithmetic mean** (average) of a set of values is the sum of the values divided by the number of values.

For example, the average of the set of values 1, 3, 5, 10, and 21 is 8 because

$$\frac{1+3+5+10+21}{5} = \frac{40}{5} = 8$$

If the average of a set of values is given, their sum can be calculated by multiplying the average by the number of values.

Example: Danny bought 5 items at an average price of 10 shekels. How much did Danny pay for all of the items?

If we multiply the average by the number of items, we will obtain $10 \cdot 5 = 50$. Thus, Danny paid a total of 50 shekels for all of the items that he bought.

In general, the term "average" will be used in the questions rather than "arithmetic mean."

A **weighted average** is an average that takes into account the relative weight of each of the values in a set.

Example: Robert's score on the mid-term exam was 75, and his score on the final exam was 90. If the weight of the final exam is twice that of the mid-term exam, what is Robert's final grade in the course?

The set of values in this case is 75 and 90, but each has a different weight in Robert's final grade for the course. The score of 75 has a weight of 1 and the score of 90 has a weight of 2. To calculate the weighted average, multiply each score by the weight assigned to it, and divide by the sum of the weights: $\frac{1 \cdot 75 + 2 \cdot 90}{1+2} = 85$. Thus, Robert's grade in the course is 85.

This calculation is identical to the calculation of a simple average of the three numbers 75, 90 and 90.

POWERS AND ROOTS

Raising a number to the nth power (n is a positive integer) means multiplying it by itself n times.

For example: $2^3 = 2 \cdot 2 \cdot 2$

Or in general: $\underbrace{a \cdot ... \cdot a \cdot a}_{n \text{ times}} = a^n$

The expression a^n is called a power; n is the exponent; and a is the base.

A positive number raised to the 0th power equals 1. Thus, for any $a \neq 0$, $a^0 = 1$.

When a number is raised to a negative power, the result is the same as that obtained by raising the reciprocal of the base to the opposite power. For example: $2^{-3} = \left(\frac{1}{2}\right)^3 = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

Or in general: $a^{-n} = \left(\frac{1}{a}\right)^n = \frac{1}{a^n}$

The nth root of a positive number a, expressed as $\sqrt[n]{a}$, is b, which if raised to the nth power, will give a as follows:

For example: $\sqrt[2]{16} = 4$, because $4^2 = 16$

$\sqrt[3]{125} = 5$, because $5^3 = 125$

$\sqrt[4]{81} = 3$, because $3^4 = 81$

It should be stressed that when $\sqrt{a}$ $(0 < a)$ appears in a question, it refers to the positive root of a.

When the root is not specified, a square (2nd-order) root is intended, for example, $\sqrt{81} = \sqrt[2]{81} = 9$.

A root can also be expressed as a power in which the exponent is a fraction. This fraction is the reciprocal of the order of the root, $\sqrt[n]{a} = a^{\frac{1}{n}}$ $(0 < a)$.

Basic rules for operations involving powers (for any n and m):

Multiplication:

To multiply powers with the same base, add the exponents: $a^m \cdot a^n = a^{m+n}$

Division:

To divide a power by another power with the same base, subtract the exponent in the denominator from the exponent in the numerator: $\frac{a^m}{a^n} = a^{(m-n)}$

Note: When the powers do not have the same base, the exponents cannot be added or subtracted.

Raising to a power:

To raise a power to a power, multiply the exponents: $(a^m)^n = a^{(m \cdot n)}$

Raising a product or a quotient to a power:

$(a \cdot b)^m = a^m \cdot b^m$; $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Since roots can also be expressed as powers, the laws for solving problems involving powers can also be applied to roots.

For example, to calculate the product $\sqrt[m]{a} \cdot \sqrt[n]{a}$ $(0 < a)$, express the roots as powers: $\sqrt[m]{a} \cdot \sqrt[n]{a} = a^{\frac{1}{m}} \cdot a^{\frac{1}{n}}$

The next step is the same as when multiplying powers; in other words, add the exponents: $a^{\frac{1}{m}} \cdot a^{\frac{1}{n}} = a^{\left(\frac{1}{m}+\frac{1}{n}\right)}$

Below are a number of basic rules that apply to inequalities involving powers:

If	$0 < b < a$	and	$0 < n$	then	$b^n < a^n$
If	$0 < b < a$	and	$n < 0$	then	$a^n < b^n$
If	$1 < a$	and	$m < n$	then	$a^m < a^n$
If	$0 < a < 1$	and	$m < n$	then	$a^n < a^m$

CONTRACTED MULTIPLICATION FORMULAS

To multiply two expressions enclosed in parentheses, each of which is the sum of two terms, multiply each of the terms in the first expression by each of the terms in the second expression, then add the products.

For example: $(a + b) \cdot (c + d) = ac + ad + bc + bd$

This general formula can be used for finding the product of any two expressions, but to save time, you might want to memorize some common formulas:

$(a + b)^2 = (a + b) \cdot (a + b) = a^2 + 2ab + b^2$
$(a - b)^2 = (a - b) \cdot (a - b) = a^2 - 2ab + b^2$
$(a - b) \cdot (a + b) = a^2 - b^2$

COMBINATORIAL ANALYSIS

The Number of Results in a Multi-Stage Experiment

The number of possible results of an experiment consisting of several independent stages (that is, they do not affect each other) is the product of the number of possible results in each stage.

For example, if we toss a die and then toss a coin, what is the number of possible results of this experiment?

The number of possible results of tossing a die is 6, and the number of possible results of tossing a coin is 2. Thus, the number of possible results of this experiment is $6 \cdot 2 = 12$. One of the 12 possible results is the number 3 on the die and tails on the coin.

It makes no difference whether we first toss the die and then toss the coin, or toss both at the same time. In either case, there are 12 possible results.

Ordered Samples

An ordered sample is one in which the order of the results obtained in a multi-stage experiment is important.
Example: A basket contains 9 slips of paper, numbered 1 through 9. If 3 slips of paper are drawn at random from the basket one after another, and their numbers written in a row, a three-digit number will be obtained. How many different numbers can be obtained in this way?

To answer the question, we have to know the sampling method that is being used. In any case, the order in which the results are obtained is important; for example, the number 123 is different from the number 213.

a. **Sampling with replacement**: Each slip of paper is replaced in the basket after it is drawn, making it possible for it to be drawn again. The number of possible numbers that can be obtained each time a slip of paper is drawn from the basket is 9. Therefore, the number of three-digit numbers that can be formed is $9 \cdot 9 \cdot 9 = 729$.

b. **Sampling without replacement:** The slips of paper that were drawn from the basket are not replaced. The number of possible numbers that can be obtained when the first slip is drawn is 9; when the second slip is drawn, only 8 (since one slip has already been withdrawn from the basket); and when the third slip is drawn, 7. Thus the number of possible numbers is $7 \cdot 8 \cdot 9 = 504$.

In general, the number of possibilities for creating an ordered row of r items out of a set of n items (3 out of 9 in the above example) is:

a. n^r, if each item can be drawn more than once (sampling with replacement).

b. $n \cdot (n - 1) \cdot ... \cdot (n - r + 1)$, if each item can be drawn no more than once (sampling without replacement).

Number of Possible Arrangements (Permutations) of an Ordered Sample

The number of different possible arrangements of the 9 slips of paper, i.e., the number of possibilities for creating an ordered row of all 9 slips of paper, with each slip appearing only once ($n = r$), equals $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 = 362{,}880$.

In general, if n is the number of items in a set, then the number of possible arrangements is $1 \cdot 2 \cdot 3 \cdot ... \cdot (n-1) \cdot n$. This number is written as **n!**, and is called "n factorial."

Non-Ordered Samples

If the order of the results obtained in a multi-stage experiment is not important, the sample is a non-ordered sample. **The number of non-ordered samples equals the number of ordered samples divided by the number of possible arrangements.**

Example: A basket contains 9 pens, each of a different color. Three pens are drawn at random from the basket and not replaced. How many samples (sets) of different colored pens can be obtained? The number of ordered samples is $9 \cdot 8 \cdot 7 = 504$. The number of possible arrangements (in each sample) is $3 \cdot 2 \cdot 1 = 6$.
The number of non-ordered samples is $\frac{504}{6} = 84$.

PROBABILITY

Probability theory is a mathematical model for phenomena (experiments) the occurrence of which is not certain. Such situations can have a number of possible scenarios or outcomes. Each possible outcome is called a "simple event," and the collection of outcomes – an "event." (For the sake of brevity, we will use the term event to mean a simple event.) Each event is assigned a number from 0 to 1, which reflects the probability (likelihood) that the event will occur. The higher the probability, the greater the chance the event will occur. An event that is certain to occur has a probability of 1, and an event that has no possibility of occurring has a probability of 0.

Sometimes, each of the possible outcomes of a particular experiment has an equal probability (in other words, each of the simple events has an equal probability).

Examples of experiments of this type

The tossing of a coin: The probability of "heads" coming up is equal to the probability of "tails" coming up. This probability is $\frac{1}{2}$.
The tossing of a die: The probability of obtaining each of the numbers appearing on the faces of the die is $\frac{1}{6}$.
These are cases of tossing a fair die/fair coin.

The random removal of a ball from a bag containing 5 balls of equal size: The probability of randomly removing each of the balls is $\frac{1}{5}$.

When all possible outcomes have an equal probability, the probability of an outcome occurring is calculated as follows:
The number of possible outcomes of a particular event, divided by the total number of possible outcomes of the experiment (phenomenon).

For example, the probability that in tossing a single die we will obtain the event "the outcome is less than or equal to 3" is $\frac{3}{6}$ or $\frac{1}{2}$, because this event has 3 possible outcomes (outcomes 1, 2 and 3), and the experiment of tossing a die has a total of 6 possible outcomes.

The probability that two events will occur

When two events occur at the same time or one after the other, two situations are possible:

A. The events are independent, i.e., the probability of one event occurring is not affected by the probability of the other event occurring.

The probability of both events occurring is equal to the product of the probabilities of each individual event occurring.

For example, in tossing two fair dice, the probability that a number that is less than or equal to 3 will turn up twice is equal to the product of the probabilities of a number that is less than or equal to 3 turning up in each of the tosses, since the outcome of tossing one die does not affect the outcome of tossing the other die.

This probability is equal to $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$.

B. The events are dependent, that is, the probability of a particular event occurring is affected by the occurrence of a different event. In other words, the probability of a particular event occurring after (or given that) we know that another event has occurred is different from the probability of that particular event occurring without such knowledge. The probability of the event "the outcome is less than or equal to 3" (we will call this event A), given that we know that in tossing the die the event "outcome is even" has occurred (we will call this event B), is calculated as follows: The probability of A occurring is the number of outcomes in which both A and B occurred (in the example, 2 is the only outcome that is both even and less than or equal to 3), divided by the number of outcomes in which B occurred (outcomes 2, 4 and 6 are even).

Therefore, the probability is $\frac{1}{3}$.

This probability is different from the probability of event A without knowledge of condition B (which equals $\frac{1}{2}$, as calculated previsouly).

Distance, Speed (Rate), Time

The speed (rate) at which an object travels is the distance that the object covers in a unit of time. The formula for the relationship between the speed, the distance the object covers and the amount of time it requires to cover that distance is:

$v = \frac{s}{t}$ where v = the speed (rate)

s = distance

t = time

All possible relationships between distance, speed and time can be derived from this formula:

$t = \frac{s}{v}$ and $s = v \cdot t$

Using these relationships, any unknown variable out of the three can be calculated if the other two variables are known. For example, a train traveled 240 kilometers at a speed of 80 kilometers per hour. How long did the journey take?

You are given v (80 kph) and s (240 km), and you have to determine t. Substituting the given information into the formula $t = \frac{s}{v}$, we get $t = \frac{240}{80} = 3$. Thus, the journey took 3 hours.

Meters can be converted to kilometers and seconds to hours, and vice versa. There are 1,000 meters in every kilometer (1 meter = $\frac{1}{1{,}000}$ kilometer).

There are 3,600 seconds, which equal 60 minutes, in every hour (1 second = $\frac{1}{3{,}600}$ hour).

A speed of 1 kilometer per hour is equal to a speed of $\frac{5}{18}$ meters per second (or $\frac{1{,}000}{3{,}600}$ meters per second).

A speed of 1 meter per second is equal to a speed of 3.6 kilometers per hour.

Work (Output)

Output is the amount of work per unit of time.
The formula for the relationship between output, amount of work and the time needed to do the work is $p = \frac{w}{t}$, where p = output (rate)
w = amount of work
t = time

All possible relationships between output, amount of work and time can be derived from this formula:
$t = \frac{w}{p}$ and $w = p \cdot t$

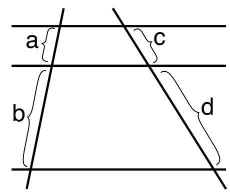
This formula can be used to calculate any unknown of the three variables if the other two are known.
For example, a builder can finish building one wall in 3 hours. How many hours would be needed for two builders working at the same rate to finish building 5 walls?

We are given the amount of work of one builder (1 wall), and the amount of time spent working (3 hours). Therefore his output is $\frac{1}{3}$ of a wall in an hour. Since the question involves two builders, the output of both is $2 \cdot \frac{1}{3} = \frac{2}{3}$

We are also given the amount of work which both builders are required to do – 5 walls. We can therefore calculate the amount of time they will need: $t = 5 : \frac{2}{3} = 5 \cdot \frac{3}{2} = \frac{15}{2} = 7\frac{1}{2}$. Thus, they will need $7\frac{1}{2}$ hours.

PARALLEL (STRAIGHT) LINES

Parallel lines that intersect any two straight lines divide the straight lines into segments that are proportional in length.
Thus in the figure, $\frac{a}{c} = \frac{b}{d}$, $\frac{a}{b} = \frac{c}{d}$ and $\frac{a}{a+b} = \frac{c}{c+d}$.
Other relationships between the segments can be deduced based on the given relationships.

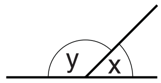


Angles

An angle is a right angle if it measures 90°.
An angle is an acute angle if it measures less than 90°.
An angle is an obtuse angle if it measures more than 90°.

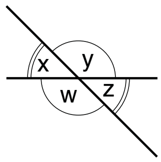
Adjacent Angles

The two angles that are formed between a straight line and a ray extending from a point on the straight line are called adjacent angles. Together they form a straight angle and their sum therefore equals 180°. For example, in the figure, x and y are adjacent angles; therefore, $x + y = 180°$.



Vertical Angles

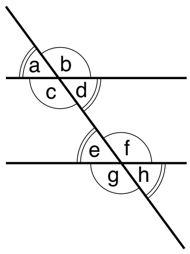
When two straight lines intersect, they form four angles. Each pair of non-adjacent angles are called vertical angles and they have the same measure.



In the figure, x and z are vertical angles and therefore have the same measure, as do y and w; in other words, $x = z$ and $y = w$.

When a straight line intersects two parallel lines (transversal), eight angles are formed, as in the figure: a, b, c, d, e, f, g, h .

Corresponding Angles



Corresponding angles are angles located on the same side of a transversal and on the same side of the parallel lines. Corresponding angles have the same measure (see figure).

Thus, in the figure, $a = e$, $c = g$, $b = f$, $d = h$

Alternate Angles

Alternate angles are located on opposite sides of a transversal and on opposite sides of the parallel lines. Alternate angles have the same measure.

Thus, in the figure, $c = f$, $d = e$, $a = h$, $b = g$.

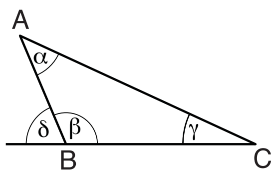
Other relationships between the different angles can be deduced based on the given relationships.

For example: since c and d are adjacent angles ($c + d = 180°$), and since c and f are alternate angles ($c = f$), then obviously, $d + f = 180°$. Similarly, we can prove that $c + e = 180°$, and so on.

TRIANGLES

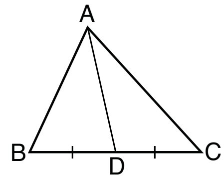
Angles of a Triangle

The sum of the interior angles of any triangle is 180°. In the figure, $\alpha + \beta + \gamma = 180°$.



An angle adjacent to one of the triangle's angles is called an exterior angle, and it is equal to the sum of the other two angles of the triangle. For example, in the figure, δ is the angle adjacent to β, and therefore $\delta = \alpha + \gamma$.

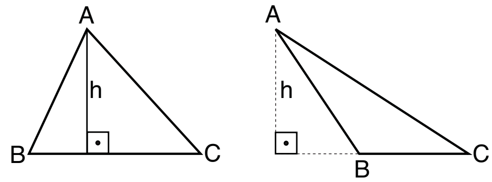
In all triangles, the longer side lies opposite the larger angle. For example, in the figure on the previous page, if $\gamma < \alpha < \beta$, it follows that side AC (which is opposite angle β) is longer than side BC (which is opposite angle α), and side BC is longer than side AB (which is opposite angle γ).



Median of a Triangle is a line that joins a vertex of a triangle to the midpoint of the opposite side.
For example, in the triangle in the figure, AD is the median to side BC (therefore BD = DC).

Altitude of a Triangle

The altitude to a side of a triangle is a perpendicular line drawn from a vertex of the triangle to the opposite side.
For example, in each of the triangles in the figures, h is the altitude to side BC.



Area of a Triangle

The area of a triangle equals half the product of the length of one of the sides multiplied by the altitude to that side.
For example, the area of each triangle ABC in the above figures is $\frac{BC \cdot h}{2}$.

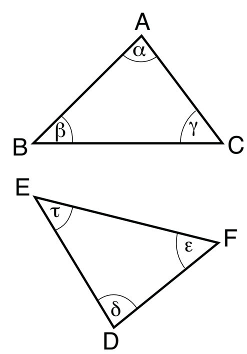
Inequality in a Triangle

In every triangle, the sum of the lengths of any two sides is greater than the length of the third side.
For example, in the triangles in the above figures (AB + BC) > AC.

Congruent Triangles

Two geometric figures are congruent if one of them can be placed on the other in such a way that they both coincide. **Congruent triangles** are a specific case of congruence.

If two triangles are congruent, their respective sides and angles are equal.
For example, triangles ABC and DEF in the figure are congruent. Therefore, AB = DE, BC = EF, AC = DF, and $\alpha = \delta$, $\beta = \tau$, $\gamma = \varepsilon$.



There are 4 theorems that enable us to deduce that two triangles are congruent:

(a) Two triangles are congruent if two sides of one triangle equal the two corresponding sides of another triangle, and the angle between these sides in one triangle is equal to the corresponding angle in the other triangle.

For example, the triangles in the figure are congruent if AB = DE, AC = DF and $\alpha = \delta$.

(b) Two triangles are congruent if two angles of one triangle are equal to the two corresponding angles of another triangle, and the length of the side between these angles in one triangle equals the length of the corresponding side in the other triangle.
For example, the two triangles in the figure are congruent if $\alpha = \delta$, $\beta = \tau$ and AB = DE.

(c) Two triangles are congruent if the three sides of one triangle equal the three sides of the other triangle.

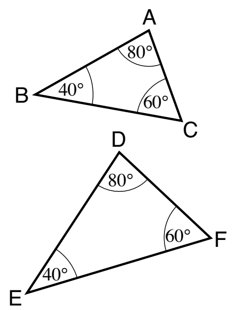
(d) Two triangles are congruent if two sides of one triangle equal the corresponding two sides of the other triangle, and the angle opposite the longer of the two sides of one triangle is equal to the corresponding angle in the other triangle.
For example, the triangles in the figure are congruent if AB = DE, AC = DF, and $\gamma = \varepsilon$ (when AB > AC and DE > DF).

Similar Triangles

Two triangles are similar if the three angles of one triangle are equal to the three angles of the other triangle. In similar triangles, the ratio between any two sides of one triangle is the same as the ratio between the corresponding two sides of the other triangle.

For example, triangles ABC and DEF in the figure are similar. Therefore, $\frac{AB}{AC} = \frac{DE}{DF}$ and so on.

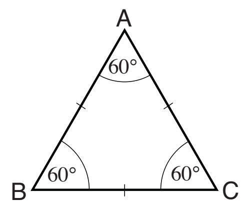
It follows that $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$.



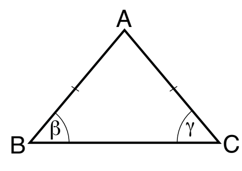
TYPES OF TRIANGLES

An **equilateral triangle** is a triangle whose sides are all of equal length. For example, in the figure, AB = BC = AC. In a triangle of this type, all of the angles are also equal (60°).

If the length of the side of such a triangle is a, then its altitude is $a\frac{\sqrt{3}}{2}$ and its area is $a^2\frac{\sqrt{3}}{4}$



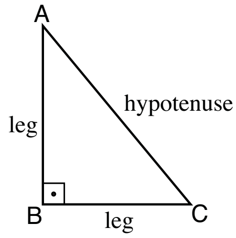
An **isosceles triangle** is a triangle with two sides of equal length.
For example, in the figure, AB = AC.
The two angles opposite the equal sides are also equal.
For example, in the figure, $\beta = \gamma$.



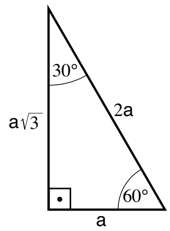
An **acute triangle** is a triangle in which all angles are acute.

An **obtuse triangle** is a triangle with one obtuse angle.

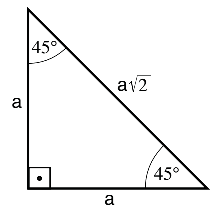
A **right triangle** is a triangle with one angle that is a right angle (90°). The side opposite the right angle (side AC in the figure) is called the **hypotenuse**, and the other two sides are the **legs** (sides AB and BC in the figure). According to the Pythagorean theorem, in a right triangle the square of the hypotenuse is equal to the sum of the squares of the legs.
For example, in the figure, $AC^2 = AB^2 + BC^2$. This formula can be used to find the length of any side if the lengths of the other two sides are given.



In a right triangle whose angles measure 30°, 60° and 90°, the length of the leg opposite the 30° angle equals half the length of the hypotenuse.
For example, the length of the hypotenuse in the figure is 2a. Therefore, the length of the leg opposite the 30° angle is a. It follows from the Pythagorean theorem that the length of the leg opposite the 60° angle is $a\sqrt{3}$.

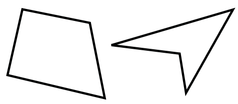


In an isosceles right triangle, the angles measure 45°, 45° and 90°, the two legs are of equal length, and the length of the hypotenuse is $\sqrt{2}$ times greater than the length of the legs.



QUADRILATERALS

A quadrilateral is any four-sided polygon. For example:



TYPES OF QUADRILATERALS

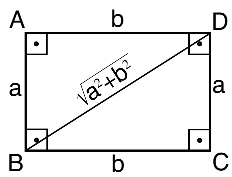
Rectangles and Squares

A **rectangle** is a quadrilateral whose angles are all right angles. The opposite pairs of sides in a rectangle are equal in length.

The **perimeter of the rectangle** in the figure is 2a + 2b or 2(a + b).

The length of the diagonal of a rectangle is $\sqrt{a^2 + b^2}$ (based on the Pythagorean theorem).

The area of the rectangle (S) is the product of the lengths of two adjacent sides. For example, in the figure, $S = a \cdot b$.

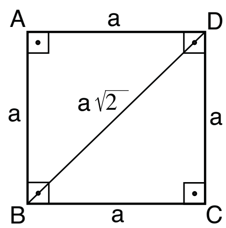


A **square** is a rectangle whose sides are all of equal length.

The **perimeter of the square** in the figure is 4a.

The length of the diagonal of the square in the figure is $\sqrt{a^2 + a^2} = a\sqrt{2}$

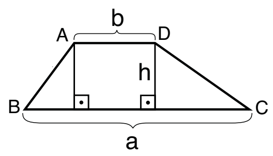
The area of a square is equal to the square of the length of the side.
For example, in the figure, $S = a^2$.



TRAPEZOID

A trapezoid is a quadrilateral with only one pair of parallel sides. The parallel sides are called **bases**, and the other two sides are called **legs**. The bases of a trapezoid are not equal, and are therefore referred to as the long base and short base. The altitude of a trapezoid is a segment joining the bases of the trapezoid and perpendicular to them.

The **area of a trapezoid** is equal to the sum of the length of the bases multiplied by half the altitude.



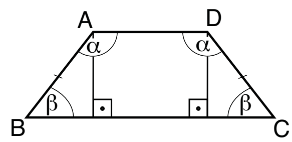
For example, in the figure: The length of the long base (BC) is a.

The length of the short base (AD) is b.

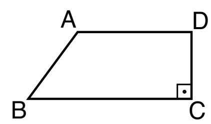
The length of the altitude is h.

The area of the trapezoid is $S = \frac{h \cdot (a+b)}{2}$.

An **isosceles trapezoid** is a trapezoid whose legs are of equal length. For example, in the figure, $AB = DC$. The base angles of an isosceles trapezoid are equal.
For example, in the figure, $\sphericalangle BAD = \sphericalangle CDA = \alpha$, $\sphericalangle ABC = \sphericalangle DCB = \beta$.
In this type of trapezoid, if two altitudes are drawn from the ends of the short base to the long base, a rectangle and two congruent right triangles are obtained.

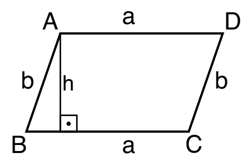


A **right trapezoid** is a trapezoid in which one of the base angles is a right angle (see figure).



PARALLELOGRAMS AND RHOMBUSES

A **parallelogram** is a quadrilateral in which each pair of opposite sides is parallel and of equal length.



For example, in the parallelogram in the figure: $AB \parallel DC$, $AD \parallel BC$
$AB = DC$, $AD = BC$

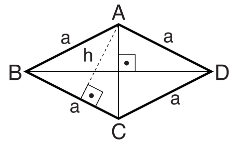
The **diagonals of a parallelogram** bisect each other.

As stated, each pair of opposite sides in a **parallelogram** is of equal length. Therefore, the **perimeter of the parallelogram** in the figure is $2a + 2b$.

The **area of a parallelogram** equals the product of a side multiplied by the altitude to that side. For example, the area of the parallelogram in the figure is $a \cdot h$.

A **rhombus** is a quadrilateral whose four sides are all equal. Each pair of opposite sides in a rhombus is parallel, and it can therefore be regarded as a parallelogram with equal sides.

Diagonals of a Rhombus



Since a rhombus is a type of parallelogram, its diagonals bisect each other. In a rhombus, the diagonals are also **perpendicular** to each other. Since all of the sides of a rhombus are of equal length, the **perimeter of the rhombus** in the figure is $4a$.

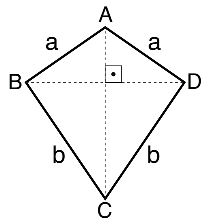
Area of a Rhombus

Since a rhombus is a type of parallelogram, its area too can be calculated as the product of a side multiplied by the altitude.
For example, the area of the rhombus in the figure is $a \cdot h$.

In addition, the area of a rhombus can be calculated as half the product of its diagonals.

For example, the area of the rhombus in the figure is $\frac{AC \cdot BD}{2}$.

KITE (DELTOID)



A kite is a quadrilateral formed by two isosceles triangles joined at their bases. For example, kite $ABCD$ in the figure is composed of triangles ABD and BCD ($AB = AD$, $CB = CD$).

The diagonal joining the vertices of the two isosceles triangles bisects the diagonal that is the base of the two isosceles triangles and is vertical to it. (For example, in the figure, AC bisects BD and $AC \perp BD$)

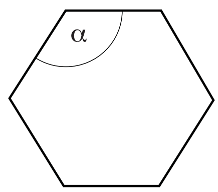
The **perimeter of the kite** in the figure is $2a + 2b$.

The **area of a kite** equals half the product of the lengths of the diagonals.

For example, the area of the kite in the figure is $\frac{AC \cdot BD}{2}$.

REGULAR POLYGON

A regular polygon is a polygon whose sides are all of equal length and whose interior angles have the same measure.



Examples: A regular pentagon is a five-sided regular polygon.
A regular hexagon is a six-sided regular polygon.
A regular octagon is an eight-sided regular polygon.

The size of the interior angle of a regular polygon with n sides can be calculated using the formula $\alpha = \left(180^\circ - \frac{360^\circ}{n}\right)$

For example, the figure shows a regular hexagon. The size of each of its interior angles is 120°, because $\alpha = 180^\circ - \frac{360^\circ}{6} = 120^\circ$

CIRCLE

A **radius** is a line segment that joins the center of a circle to a point on its circumference.

A **chord** of a circle is a line segment that passes though the circle and joins two points on its circumference.

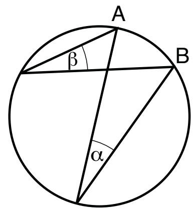
A **diameter** of a circle is a chord that passes through its center. The length of a circle's diameter is twice the length of its radius. If the radius of a circle is r, the diameter of the circle is $2r$.

An **arc** is the part of the circle between two points on its circumference.

The **circumference** of a circle with radius r is $2\pi r$. (The value of π is approximately 3.14.)

The **area** of a circle with radius r is πr^2.

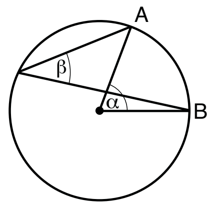
Inscribed Angle



An inscribed angle is an angle whose vertex lies on the circumference of a circle and whose sides are chords of the circle. Inscribed angles intercepting the same arc have the same measure.

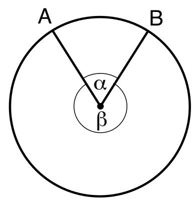
For example, in the figure, angles α and β are inscribed angles, both of which intercept arc AB; therefore, $\alpha = \beta$. An inscribed angle that lies on the diameter of a circle (that is, on an arc whose length equals half the circle's circumference) is a right angle.

Central Angle



A central angle is an angle whose vertex is the center of the circle and whose sides are radii of the circle (in the figure, α is a central angle). A central angle is twice the size of any inscribed angle that intercepts the same arc. For example, in the figure, α is a central angle and β is an inscribed angle, and both intercept the same arc AB. Therefore, $\alpha = 2\beta$.

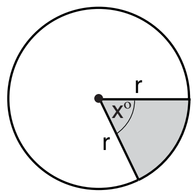
Arc



Two points on the circumference of a circle define two arcs. For example, in the figure, points A and B define two arcs – one corresponding to central angle α and one corresponding to central angle β. The smaller arc AB corresponds to α, the smaller of the two angles.

The length of this arc is $2\pi r \cdot \frac{\alpha}{360}$ (r is the radius of the circle).

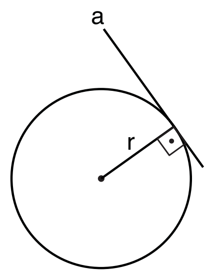
Sector



A sector is the part of a circle bounded by two radii and an arc.
The angle between the two radii is a central angle.

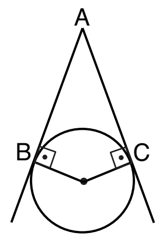
For example, the shaded region in the figure is the sector of a circle with central angle α. The area of the sector of the circle is $\pi r^2 \cdot \frac{x}{360}$

Tangent to a Circle



A tangent is a line that touches the circumference of a circle at only one point, the "point of tangency." The angle formed by the radius and the tangent at that point is a right angle.
For example, in the figure, line segment a is tangent to the circle with radius r.

Two Tangents to a Circle



Two tangents to a circle that intersect at a (particular) point are also called two tangents that originate at one point.

The length of each tangent is the length of the segment that joins the tangents' point of intersection and the point of tangency.

Tangents to a circle that originate at one point are equal in length.
For example, in the figure, A is the point of intersection, B and C are the points of tangency, and AB = AC.

Polygon Circumscribing a Circle

A polygon that circumscribes a circle is a polygon whose sides are all tangent to the circle.

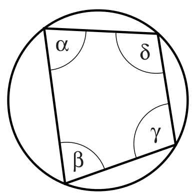
Polygon Inscribed in a Circle

A polygon inscribed in a circle is a polygon whose vertices all lie on the circumference of the circle.

Inscribed Triangle

Every triangle can be inscribed in one and only one circle (that is, a circle with the vertices of the triangle lying on its circumference). If the inscribed triangle is a right triangle, the center of the circle that circumscribes it is the midpoint of the triangle's hypotenuse.

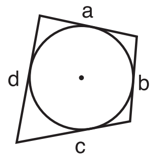
Quadrilateral Inscribed in a Circle



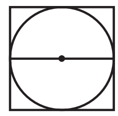
Not every quadrilateral can be inscribed in a circle. The sum of the opposite angles of a quadrilateral inscribed in a circle always equals 180°.
For example, in the quadrilateral in the figure, $\alpha + \gamma = 180°$
$\beta + \delta = 180°$

Quadrilateral Circumscribing a Circle

When a quadrilateral circumscribes a circle, the sum of the lengths of each pair of opposite sides is equal.
For example, in the quadrilateral in the figure, $a + c = b + d$.

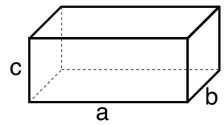


When a square circumscribes a circle, the length of the side of the square equals the length of the diameter of the circle (see figure).



SOLIDS

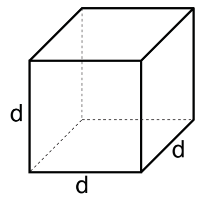
Box (Rectangular Prism) and Cube



A **box** is a three-dimensional figure with six rectangular faces. The box's three dimensions are its length, width and height (a, b and c respectively, in the figure).

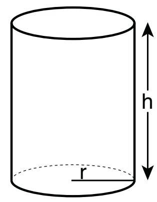
The **surface area** of a box is the sum of the areas of its faces. The surface area of the box in the figure is $ab + ac + bc + ab + ac + bc$ or $2ab + 2ac + 2bc$.

The **volume** (V) of a box is the product of its length, width and height.
The volume of the box in the figure is $V = a \cdot b \cdot c$.
A **cube** is a box whose three dimensions are all equal.
All of the faces of a cube are equal in area.
The area of each face of the cube in the figure is d^2.
Therefore, the **surface area** of the cube is $6d^2$.
The **volume** of the cube in the figure is $V = d^3$.



Cylinder

A cylinder is a three-dimensional figure whose two bases are congruent circles on parallel planes. In a right cylinder, the line joining the centers of the circles is vertical to each of the bases.



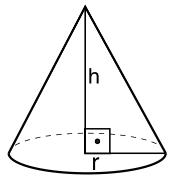
The **lateral surface area** of a cylinder with base radius of length r and with height h is the product of the circumference of the base multiplied by the height, that is, $2\pi r \cdot h$.

The **total surface area** of a cylinder is the sum of the areas of the bases and the lateral surface. The area of each base is πr^2 and the lateral surface area is $2\pi r \cdot h$. Thus, the total surface area is $2\pi r \cdot h + 2\pi r^2 = 2\pi r \cdot (h + r)$.

The **volume** of a cylinder is the product of the area of one of the bases multiplied by the height, that is, $\pi r^2 \cdot h$.

Cone

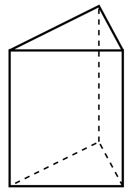
A cone is a figure formed by joining the points on the circumference of a circle with a point outside the plane of the circle.



A **right cone** is formed when the point outside the circle lies on a line that passes through the center of the circle and is perpendicular to the plane of the circle.

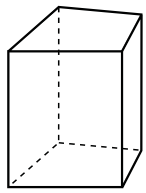
The **volume** of a cone with base radius r and height h is $V = \frac{\pi r^2 \cdot h}{3}$.

Right Prism



A right prism is a three-dimensional figure whose two bases are congruent polygons on parallel planes and whose lateral faces are rectangles. The type of prism is defined by the number of sides of its base. For example, a triangular prism has three-sided bases, a quadrangular prism has four-sided bases, and so on (see figures).

The **height** of a prism is the length of the segment that joins the bases and is perpendicular to them. It is the distance between the bases of the prism.

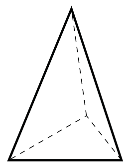


The **lateral surface area** of the prism is the sum of the areas of all the lateral faces. The lateral surface can also be calculated by multiplying the perimeter of the prism's base by its height.

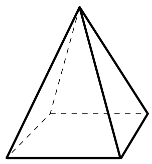
The **total surface area** of a prism is the sum of the lateral surface area and the areas of the two bases.

The **volume** of a prism equals the area of one of the bases multiplied by the height.

Pyramid

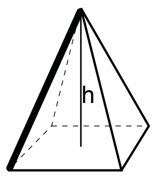


A pyramid is a figure formed by joining the vertices of any polygon to a point outside the plane of the polygon called the vertex or apex of the pyramid. The polygon is called the base of the pyramid.



The lateral faces of the pyramid are triangles. A pyramid is referred to by the number of sides of its base. For example, a triangular pyramid has a three-sided base, a quadrangular pyramid has a four-sided base, and so on (see figure).

The **height** of a pyramid is the line segment extending perpendicularly from the pyramid's vertex to its base. This is the distance between the pyramid's vertex and base (see figure).



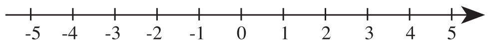
If S is the area of the pyramid's base and h is the pyramid's height, then the pyramid's **volume** is $V = \frac{S \cdot h}{3}$

Edge

The edge of a three-dimensional figure is the straight line formed where two faces meet. For example, a box has 12 edges. The bold line in the pyramid on the previous page is one of its edges.

NUMBER LINE (AXIS)

A number line is a geometric representation of the relationships between numbers.

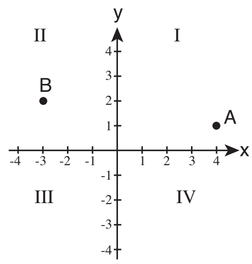


* The numbers along the axis increase to the right.

* The distance between points along the axis is proportional to the difference between the numerical values corresponding to the points. For example, the distance between the points corresponding to values (-4) and (-2) is equal to the distance between the points corresponding to values 3 and 5.

Cartesian Coordinate System

Cartesian coordinates on a coordinate plane have two number lines (axes) that are perpendicular to each other. The horizontal line is called the x-axis and the vertical line is called the y-axis. The numbers along the x-axis increase to the right. The numbers along the y-axis increase upwards.



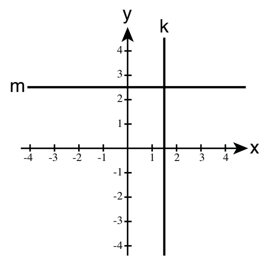
The axes divide the plane into four quadrants, designated in the figure by Roman numerals I, II, III, IV.

Each point in the coordinate plane corresponds to a pair of x and y values. For example, the x-value of point A in the figure is 4, and its y-value is 1. The x-value of point B in the figure is (-3) and its y-value is 2.

It is customary to write the x- and y-values of the points in parentheses, with the x-value to the left of the y-value, as follows: (x , y). For example, point A is written as A(4 , 1) and point B is written as B(-3 , 2).

The x- and y-values for a point are sometimes called the coordinates of that point. The point in the plane corresponding to (0 , 0) is the point of intersection of the two axes and is called the origin.

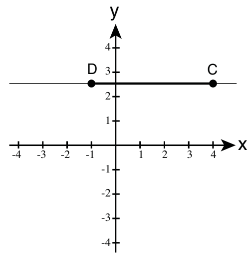
All points on a line parallel to the x-axis have the same y-coordinate, and all points on a line parallel to the y-axis have the same x-coordinate.



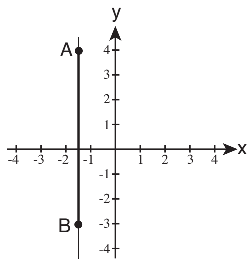
For example, in the figure, line k is parallel to the y-axis. Thus, all of the points on line k have the same x-coordinate.
In the figure, x = 1.5

Line m is parallel to the x-axis. Thus, all of the points on line m have the same y-coordinate.
In the figure, y = 2.5

Only one line can be drawn through any two points on a plane. The part of the line between the two points is called a line segment.



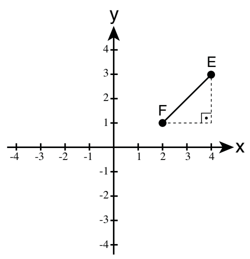
If the line segment is parallel to the x-axis, its length is the difference (in absolute value) between the x-coordinates of the points. For example, in the figure, line segment CD is parallel to the x-axis. The x-coordinate of point C is 4 and the x-coordinate of point D is (-1). The difference between the x-coordinates of the points is 5. Therefore, the length of line segment CD is 5.



If the line segment is parallel to the y-axis, its length is the difference (in absolute value) between the y-coordinates of the points.

For example, in the figure, line segment AB is parallel to the y-axis.

The y-coordinate of point A is 4 and the y-coordinate of point B is (-3). The difference between the y-coordinates of the points is 7. Therefore, the length of line segment AB is 7.



If the line segment is not parallel to one of the axes (for example, line segment EF in the figure), its length can be calculated using the Pythagorean theorem: Draw a right triangle such that the segment is the hypotenuse and the legs are parallel to the x-axis and the y-axis. The length of the leg parallel to the x-axis equals the difference between the x-coordinates of points E and F (4 – 2 = 2), and the length of the leg parallel to the y-axis equals the difference between the y-coordinates of points E and F (3 – 1 = 2).

Using the Pythagorean theorem, we can calculate the length of the hypotenuse: ($EF = \sqrt{2^2 + 2^2} = \sqrt{8}$)

QUESTIONS AND PROBLEMS

These questions cover a variety of topics, such as distance problems, work problems, combinatorial analysis and probability, equations, geometry, and so on. Some are verbal questions which have to be converted into algebraic expressions and the solution given in numerical form; some are non-verbal questions that already have the format of algebraic expressions; and some deal with characteristics of geometric shapes, such as area, volume, angles, and so on. Below are some sample questions, together with solutions and explanations.

Note: The examples in the *Guide* are arranged by type, but this is not the case in the actual exam.

VERBAL QUESTIONS

1. A driver covered a third of the distance from Haifa to Eilat at a speed of 75 kph. He covered a fifth of the **remaining** distance in one hour, and the rest of the distance at a speed of 80 kph. The distance between Haifa and Eilat is 450 kilometers. If the driver had driven the entire distance at a constant speed, at what speed would he have driven so that the journey from Haifa to Eilat would take exactly the same amount of time?
(kph = kilometers per hour)

(1) 70 kph
(2) 75 kph
(3) 80 kph
(4) None of the above

This question is a mathematical problem presented in verbal form; therefore the first step is to convert it into algebraic expressions. Start by clearly defining what you are asked to find: the **speed** at which to drive in order to cover the **distance** between Haifa and Eilat in the same amount of **time** that it took the driver in the question. Therefore, this is a distance problem, and the formula $v = \frac{s}{t}$, which connects distance, speed and time can be applied since the distance (s) is given, the time (t) can be calculated, and the speed (v) is the unknown that you have to find. The question provides the information that the distance between Haifa and Eilat is 450 kilometers. The total amount of time needed by the driver to cover the entire distance between Haifa and Eilat can be calculated as follows:
The distance is divided into three segments. The time it took the driver to cover each segment is as follows:

a. A third of the distance is **150 km**, because $450 \cdot \frac{1}{3}$ kilometers equals 150 kilometers. It took the driver **two hours** to cover this segment, because it takes two hours to travel 150 kilometers at a speed of 75 kph $\left(\frac{150}{75} = 2\right)$.

b. A fifth of the remaining distance is **60 kilometers**, since the remaining distance is $450 - 150 = 300$ kilometers, and $300 \cdot \frac{1}{5}$ kilometers equals 60 kilometers.
The question provides the information that the driver covered this segment of the journey in **one hour**.

c. The rest of the distance is **240** kilometers, since 450 – 150 – 60 = 240. The driver covered this distance in **three hours**, because it takes three hours to travel 240 kilometers at a speed of 80 kph.

Thus, the journey from Haifa to Eilat took a total of **6 hours** (two hours, plus one hour, plus three hours). By substituting the data into the formula, it is now possible to compute the constant speed at which it is necessary to drive in order to cover 450 km in 6 hours: $t = 6$, $s = 450$, $v = \frac{s}{t} = \frac{450}{6} = 75$.

Thus, the speed is 75 kph, and the correct response is (2).

2. At the age of 10 days, a baby elephant eats 5 candies. From this age onwards, its appetite grows, and each day it eats twice the number of candies it ate the previous day.

How many candies will the baby elephant eat at the age of 14 days?

(1) 40
(2) 80
(3) 100
(4) 120

On the tenth day, the baby elephant eats 5 candies. Each day after that it eats twice the number of candies that it ate the previous day. Thus, on the 11th day it eats 10 candies ($5 \cdot 2$), on the 12th day it eats 20 candies ($5 \cdot 2 \cdot 2$), and so on. In general, if n is a positive integer, then on day ($10 + n$) the baby elephant will eat $5 \cdot 2^n$ candies.

Thus, on the 14th day it will eat 80 candies ($5 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 5 \cdot 2^4 = 80$), and the correct response is (2).

3. A restaurant offers 3 different first courses and 4 different main courses. In addition to the first course and the main course, it also offers a choice of soup or dessert. How many different combinations of three-course meals can be put together at this restaurant?

(1) 12
(2) 14
(3) 18
(4) 24

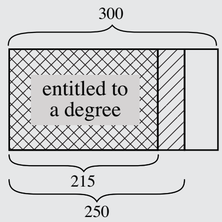
There are **three** possible choices for the first course, and **four** different main courses that can be added to **each** first course chosen. Thus, there are $4 \cdot 3$ different combinations of first courses and main courses. To each of these 12 combinations, either soup or dessert can be added. In other words, there are a total of $12 \cdot 2$ different combinations of the three courses, which equals 24 different possibilities. The correct response is therefore (4).

4. Students receive a B.A. degree only after passing all their tests and submitting all their papers. Out of 300 students, 250 passed all their tests and 215 submitted all their papers. How many students received a B.A. degree?

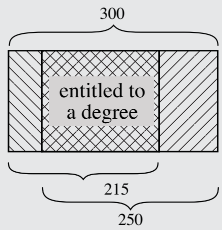
(1) at least 215
(2) no more than 185
(3) exactly 215
(4) at least 165

The question deals with two groups of students: those who submitted all their papers and those who passed all their tests. The students belonging to both groups are the ones entitled to a degree. The amount of overlap between the two groups is not known, but there are two possible extremes. We will use a diagram to illustrate them:

– In a case of **maximum overlap** of the two groups, the maximum number of students would be entitled to a degree. There is maximum overlap when all 215 students who submitted all their papers also passed all their tests. In other words, **at most** 215 students would be entitled to a degree.



– In a case of **minimum overlap** of the two groups, the minimum number of students would be entitled to a degree. When each student not entitled to a degree has only one reason for this, there is minimal overlap. This gives the maximum number of students not entitled to a degree. Fifty students (300 – 250) were not entitled to a degree because they did not pass all their tests, and 85 students (300 – 215) were not entitled to a degree because they did not submit all their papers. In other words, the maximum number of students who would not be entitled to a degree is 50+85=135. Thus, the minimal number of students entitled to a degree is 300 – 135 = 165. In other words, **at least** 165 students are entitled to a degree.



Hence, the number of students entitled to a degree could range from 165 to 215. The correct response is therefore (4).

5. A factory manufacturing at a steady rate produces 20 cars in 4 days. How many cars could 3 such factories produce in 6 days, if they work at the same rate?

(1) 60
(2) 80
(3) 90
(4) 120

This is a work problem. One way of solving such problems is by determining the output of one work unit (in this case one factory) per one time unit (in this case one day), and then multiplying by the number of work units (3 factories) and by the required number of time units (6 days). Thus, if a factory produces 20 cars in 4 days, then it produces 5 cars per day (20 : 4 = 5). Therefore, in 6 days, 3 factories will produce 5 · 6 · 3 cars, which equals 90 cars. The correct response is (3).

6. There are 20 white hats and 13 black hats in a box. Jack drew 3 black hats in succession from the box, without replacing them.
What is the probability that the fourth hat that he draws at random will also be black?

(1) $\frac{13}{33}$

(2) $\frac{10}{33}$

(3) $\frac{1}{3}$

(4) $\frac{1}{33}$

You have to calculate the probability of Jack drawing a black hat after three black hats are drawn. The probability is the number of black hats remaining in the box divided by the total number of hats (black or white) remaining in the box. After three black hats were drawn from the box, 10 black hats and 20 white hats remained in the box. In other words, out of the 30 hats in the box, 10 are black. Thus, the probability of Jack now drawing a black hat is $\frac{10}{30}$, which is $\frac{1}{3}$.

Therefore, the correct response is (3).

NON-VERBAL QUESTIONS

1. Given: $2^x \cdot 2^y = 32$
$x + y = ?$

(1) It is impossible to determine from the information given.
(2) 5
(3) 8
(4) 4

According to the laws of exponents, when multiplying powers with the same base, we add the exponents. Therefore, $2^x \cdot 2^y = 2^{x+y}$, and according to the information provided, $2^{x+y} = 32$.
In order to find the value of $x + y$, we have to express 32 as a power of base 2. The exponent of this power will equal $x + y$. 32 as a power of 2 is expressed as $32 = 2^5$. It follows that $2^{x+y} = 2^5$.
When two equal powers have the same base, their exponents are also equal, and we can therefore deduce that $x + y = 5$.
Thus, the correct response is (2).

2. The average of the three numbers x, y, and z is $x \cdot y$.

What does z equal?

(1) $3 \cdot x \cdot y - x - y$
(2) $x \cdot y - x - y$
(3) $3 \cdot x \cdot y + x + y$
(4) $3 \cdot x \cdot y - (x - y)$

An average (arithmetic mean) is the sum of the terms divided by the number of terms.

Thus, the average of x, y, and z equals $\frac{x+y+z}{3}$. Substitute the information in the question into the equation: $\frac{x+y+z}{3} = x \cdot y$; multiply both sides by 3: $x + y + z = 3 \cdot x \cdot y$; and solve for z: $z = 3 \cdot x \cdot y - x - y$.

Thus, the correct response is (1).

---

3. Given: $\frac{a+b}{2} = 9$ and $\frac{c+d+e}{3} = 4$

What is the value of the expression $\frac{a+b+c+d+e}{5}$?

(1) 5
(2) 6
(3) 6.5
(4) 13

Let us simplify the two given equations:

Multiplying both sides of the equation $\frac{a+b}{2} = 9$ by 2 gives us $a + b = 18$.

Multiplying both sides of the equation $\frac{c+d+e}{3} = 4$ by 3 gives us $c + d + e = 12$.

We can now add the results: $a + b + c + d + e = 18 + 12 = 30$, which is the numerator of the expression whose value you are asked to find.

Thus, the value of the expression you are asked to solve is $\frac{30}{5} = 6$, and the correct response is (2).

4. Given: $B < C$
$B < D < A$

Which of the following expressions is **necessarily** true?

(1) $C < D$
(2) $D < C$
(3) $C < A$
(4) None of the above expressions is necessarily true.

From the information provided it is impossible to make any deductions about the relationship of C to A and D. For example, the two situations below hold true because they do not contradict the provided information:

(a) $B < C < D < A$
(b) $B < D < A < C$

The expression in (1) is true in situation (a) but not in situation (b). The expression in (2) is true in situation (b) but not in situation (a). The expression in (3) is true in situation (a) but not in situation (b). Thus, each of the expressions is true in certain situations and not true in others. Therefore, none of the expressions is **necessarily** true, and the correct response is (4).

5. K is an even number and P is an odd number.
Which of the following statements is <u>not</u> correct?

(1) $P - K - 1$ is an odd number
(2) $P + K + 1$ is an even number
(3) $P \cdot K + P$ is an odd number
(4) $P^2 + K^2 + 1$ is an even number

Let us examine each of the statements:

(1) The difference between an odd number (P) and an even number (K) is an odd number. Therefore, $P - K$ is an odd number. If we subtract 1 from the odd number that is obtained, we get an even number. Therefore, $P - K - 1$ is an even number, and the statement is **not** correct.

(2) The sum of an odd number (P) and an even number (K) is an odd number. Therefore, $P + K$ is an odd number. If we add 1 to an odd number, we get an even number.
Therefore, $P + K + 1$ is an **even** number, and the statement is **correct**.

(3) The product of an even number and any integer is always an even number; therefore, the product of $P \cdot K$ is an even number. If we add an odd number to the even product, we get an odd number. Therefore, $P \cdot K + P$ is an **odd** number and the statement is **correct**.

(4) The square of an odd number (P^2) is an odd number because it is the product of an odd number multiplied by an odd number ($P \cdot P$), and the square of an even number (K^2) is an even number because it is the product of an even number multiplied by an even number ($K \cdot K$). The sum of the two squared numbers ($P^2 + K^2$) is odd because it is the sum of an odd number and an even number. If we add 1 to this sum, we get an even number. Thus, $P^2 + K^2 + 1$ is **even**, and the statement is **correct**.

In this question, you are asked to mark the statement that is **not** correct, and therefore (1) is the correct response.

GEOMETRY

1. A liquid that fills a rectangular container whose dimensions are 2 cm x 10 cm x 20 cm is poured into a cylindrical container whose base radius is 5 cm.
What height (in cm) will the surface of the liquid reach in the cylindrical container?

(1) $\frac{16}{\pi}$

(2) $\frac{40}{\pi}$

(3) 8π

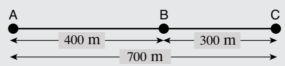
(4) 8

The volume of a rectangular container is the product of its three dimensions. Thus, the volume of the liquid in the rectangular container is $2 \cdot 10 \cdot 20$ cubic centimeters, which equals 400 cubic centimeters. After this liquid is poured into the cylindrical container, its volume does not change, but it acquires the shape of the cylinder. You must now find the height of this cylinder whose base radius is 5 centimeters and whose volume is 400 cubic centimeters. This is the height that the water will reach in the cylinder. The formula for the volume of a cylinder is $\pi r^2 \cdot h$, and you have to find h, given that $r = 5$ and the volume is 400 cubic centimeters.
Substitute the numbers into the formula: $\pi \cdot 5^2 \cdot h = 400$, that is, $\pi \cdot 25 \cdot h = 400$. To solve for h, divide the two sides by 25π: $h = \frac{16}{\pi}$. Therefore, the correct response is (1).

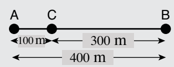
2. The distance between points A and B is 400 meters.
The distance between points B and C is 300 meters.
It follows that the distance between points A and C is **necessarily** -

(1) 100 meters
(2) 500 meters
(3) 700 meters
(4) It is impossible to determine from the information given.

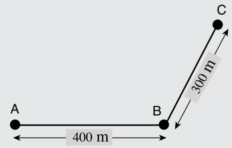
The data in this question does not provide us with information on the relative placement of the three points, and they could be arranged in many ways, such as:



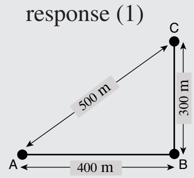
Appropriate for response (3)



Appropriate for response (1)



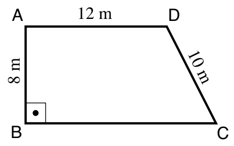
Not appropriate for any of responses (1) - (3)



Appropriate for response (2)

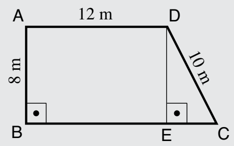
All of these placements are possible, as well as many others, and none of them is necessarily correct. Therefore, the correct response is (4).

3. The accompanying figure shows a right trapezoid ($AD \parallel BC$). Based on the information in the figure, what is the area of the trapezoid?



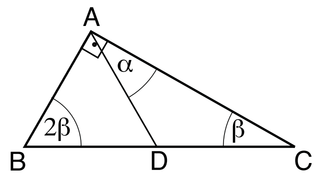
(1) 150 m^2
(2) 120 m^2
(3) 108 m^2
(4) 96 m^2

The formula for calculating the area of a trapezoid with bases a and b and height h is $S = \frac{(a+b)\cdot h}{2}$
The figure provides information on the length of the short base and the height (since this is a right trapezoid, the leg perpendicular to the bases is actually the height of the trapezoid). There is no information in the figure about the length of the long base. In order to calculate it, drop a perpendicular from point D to base BC (DE in the figure below). Rectangle $ABED$ is obtained whose length is 12 meters and width 8 meters. Thus, $BE = 12$ and $DE = 8$. It remains to calculate the length of EC in order to find the length of the trapezoid's long base. The length of segment EC can be calculated using the Pythagorean theorem for right triangle DEC: $DE^2 + EC^2 = DC^2$.
Solve for EC: $EC = \sqrt{DC^2 - DE^2}$ and substitute the information: $EC = \sqrt{10^2 - 8^2} = 6$. The length of the long base is thus 18 meters (12 meters + 6 meters). After determining the length of the long base, we can compute the area of the trapezoid: $S = \frac{(12+18)\cdot 8}{2} = 120$. The area of the trapezoid is thus 120 m^2, and the correct response is (2).



4. The accompanying figure shows right triangle ABC and isosceles triangle ABD ($AB = AD$). Based on this information and the information in the figure,

$\alpha = ?$



(1) 60°
(2) 45°
(3) 30°
(4) 25°

The sum of the angles of a triangle is 180°. Therefore, in triangle ABC we can apply the equation $90° + 2\beta + \beta = 180°$. Solving the equation, we obtain $\beta = 30°$.
We are given the information that triangle ABD is an isosceles triangle.
It follows that $\sphericalangle ADB = \sphericalangle ABD$. Since $\sphericalangle ABD = 2\beta$, then $\sphericalangle ABD = \sphericalangle ADB = 60°$.
In triangle ABD, $\sphericalangle BAD + \sphericalangle ABD + \sphericalangle ADB$ equals 180°.
Substituting the values of the angles that were calculated, $\sphericalangle BAD = 180° - 60° - 60° = 60°$.
According to the figure, $\sphericalangle BAD + \alpha = \sphericalangle BAC$. Substituting the known values of the angles, we obtain $60° + \alpha = 90°$. Thus, $\alpha = 30°$ and the correct response is (3).

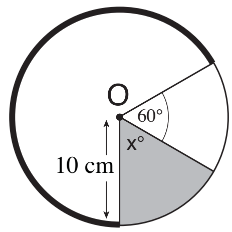
5. The accompanying figure shows a circle whose center is O and whose radius is 10 centimeters long.
The shaded region equals $\frac{1}{6}$ the area of the circle.
Based on this information and the information in the figure, what is the length (in cm) of the arc shown in bold?

(1) 30π

(2) $\frac{40}{3}\pi$

(3) $\frac{20}{3}\pi$

(4) 20π



The length of the arc shown in bold is equal to the circumference of the entire circle minus the length of the arc not in bold. To find the length of the arc not in bold, you must determine the size of the central angle that intercepts this arc. This angle consists of 60° (see figure) plus the central angle of the shaded sector. To solve the question, you must determine the size of the central angle of the shaded sector. The central angle of the shaded sector can be found by means of the formula for the area of a sector of a circle $\pi r^2 \cdot \frac{x}{360}$. In this formula, x is the central angle of the sector. It is given that the area of the shaded sector equals $\frac{1}{6}$ the area of the circle, i.e. $\frac{\pi r^2}{6}$ (since the area of the entire circle equals πr^2). Substitute this information into the formula for the sector of a circle, $\pi r^2 \cdot \frac{x}{360} = \frac{\pi r^2}{6}$, reduce the two sides by πr^2 to obtain $\frac{x}{360} = \frac{1}{6}$ and solve for x: $x = \frac{360}{6} = 60°$. Thus, the size of the angle opposite the arc not in bold is 60° + 60° = 120°. The length of the arc that intercepts this angle is $2\pi r \cdot \frac{120}{360} = 2\pi r \cdot \frac{1}{3}$, that is $\frac{1}{3}$ of the circumference of the circle. Thus, the length of the complementary arc (shown in bold), which you were asked to find, is $\frac{2}{3}$ of the circumference of the circle.
Substitute the data into the formula for the circumference of a circle: $\frac{2}{3} \cdot 2\pi r = \frac{2}{3} \cdot 2\pi \cdot 10 = \frac{40\pi}{3}$, and the correct response is (2).

6. The accompanying figure shows right triangle ABC.
BD bisects $\sphericalangle$ABC.
Based on this information and the information in the figure,

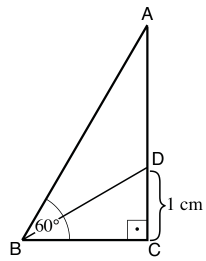
AD = ?

(1) 1 cm

(2) 2 cm

(3) $\sqrt{3}$ cm

(4) $\frac{4}{\sqrt{3}}$ cm



If we can determine the length of AC, we will be able to subtract the length of CD from it (it is given that it is 1 centimeter) and obtain the length of AD. Triangle ABC is a 30°-60°-90° triangle, because $\sphericalangle$ACB = 90° and $\sphericalangle$ABC = 60°. In a triangle of this type, AC = BC $\cdot \sqrt{3}$.
From the information that BD bisects $\sphericalangle$ABC, it follows that $\sphericalangle$DBC = 30°; therefore triangle BDC is also a 30°-60°-90° triangle.

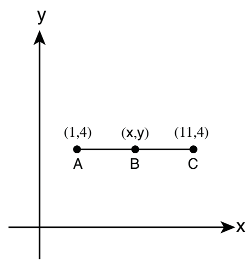
In triangle BDC, BC = CD $\cdot \sqrt{3}$. In other words, BC = $\sqrt{3}$ centimeters, and therefore AC = $\sqrt{3} \cdot \sqrt{3}$ = 3 centimeters. Subtracting the length of CD from the length of AC, we obtain AD = (3 – 1) = 2 centimeters.
Thus, the correct response is (2).

Another way of solving the problem: $\sphericalangle$BAD = 30° (based on the sum of the angles in triangle ABC). $\sphericalangle$ABD also equals 30°, because BD bisects $\sphericalangle$ABC. Therefore, triangle DAB is an isosceles triangle (AD = BD).
Triangle BDC is a 30°-60°-90° triangle, and therefore BD = CD · 2 = 2 · 1 = 2 centimeters. Thus, AD = 2 centimeters.

7. In the accompanying coordinate system, point B lies on line segment AC; AB = BC.

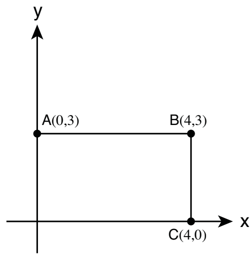
What is the x-coordinate of point B?

(1) 7
(2) 6
(3) 5
(4) 4



Line segment AC is parallel to the x-axis because the y-coordinates of points A and C are equal. Its length can be determined by calculating the difference between the x-coordinates of points C and A. Thus, the length of line segment AC is 10 (11 – 1 = 10). We are given that AB = BC. Therefore, the length of line segment AB is 5 and the x-coordinate of point B is 6 (1 + 5 = 6). The correct response is thus (2).

8. What is the area of the rectangle in the figure?



(1) 14
(2) 12
(3) 7
(4) 4

The area of a rectangle is obtained by multiplying its length by its width. Let us calculate the length of the rectangle, which is actually the length of segment AB. Its length equals the difference between the x-coordinates of points A and B, i.e., $4 - 0 = 4$. The width of the rectangle is the length of segment BC, which is equal to the difference between the y-coordinates of points B and C, i.e., $3 - 0 = 3$. Thus, the area of the rectangle is equal to $4 \cdot 3 = 12$; the correct response is (2).

QUESTIONS ON GRAPH AND TABLE COMPREHENSION

These questions involve information appearing in a graph or table. The graph or table is usually accompanied by a short explanation. In a table, the numerical data are arranged in columns and rows, whereas in a graph they are presented in graphic form, such as a curve, bar chart and so on. The questions are of two main types: questions involving **the reading of data**, in which you are asked to find information appearing in the graph or table, and **inference questions**, in which you are asked to make various inferences from the data appearing in the graph or table. Below are samples of a graph and a table, followed by questions and explanations.

GRAPH COMPREHENSION

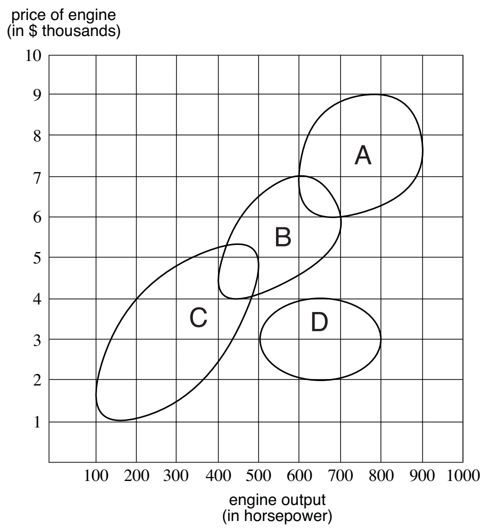
INSTRUCTIONS:

Study the graph below and answer the questions that follow.

EXPLANATION OF THE GRAPH:

The accompanying graph presents information on 4 different technologies used for producing a certain type of engine.

Each technology is represented by a letter (A–D) and by a closed area. All points within that area and on its perimeter represent the range of prices and horsepowers that are possible for that technology. For example, using technology A, a 750-horsepower engine can be manufactured at a price of $6,000-$9,000. In other words, it is possible to manufacture an engine of this type at a price of $8,500, but it is not possible to manufacture an engine with the same horsepower at a price of $5,000.



Note:

1. Technologies A and B have an area that is common to both, as do technologies B and C.
2. In answering each question, disregard the information appearing in the other questions.

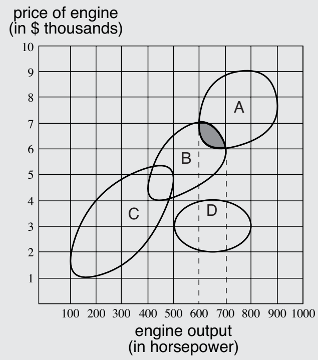
QUESTIONS AND EXPLANATIONS:

1. What is the range of engine outputs (in horsepower) that can be obtained using both technology A and technology B?

(1) 400–500
(2) 500–600
(3) 600–700
(4) None of the above

In order to solve graph comprehension questions, you must "translate" the question into the terms of the graph and then find the necessary information in the graph. The question deals with engines that can be manufactured using technology A as well as technology B. These engines are depicted in the graph by the areas of these two technologies that overlap. The overlapping area is the shaded region in figure I. You must now find the range of outputs of these engines. Since the horizontal axis represents the engine outputs, the boundaries of the overlapping area that lie on the horizontal axis represent the range of outputs of engines manufactured using both technologies. In the illustration, the area where A and B overlap is bounded on the horizontal axis by 600 and 700 horsepower. In other words, engines with an output ranging from 600 to 700 horsepower can be produced using technology A as well as technology B, and the correct response is (3).

Figure I

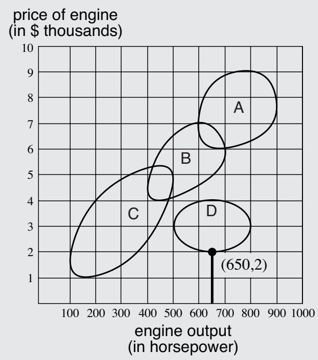


2. There is talk of producing an engine with an output of 650 horsepower.
What is the minimum price (in dollars) at which it can be manufactured?

(1) 1,000
(2) 2,000
(3) 1,500
(4) 2,500

In this question, the starting point is an engine with an output of 650 horsepower. As stated, engine output is represented on the horizontal axis of the graph. Therefore, the first step is to locate the specified output (650 horsepower) on the horizontal axis and then to find the minimum price of an engine with this output. Draw a vertical line from the point on the horizontal axis that represents 650 horsepower to a point of contact with one of the technology areas (see figure II). The lowest point of contact with one of the technology areas will represent the lowest possible price for an engine with an output of 650 horsepower. The **lowest** point of contact intercepts the boundary of the area representing technology D. This point represents a price of $2,000 on the vertical axis, and that is therefore the minimum price of an engine with the desired output. Thus, the correct response is (2).

Figure II

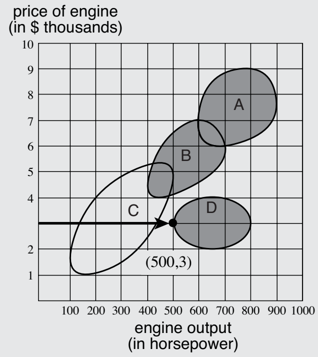


3. Due to a technical problem, it is no longer possible to produce engines using technology C. What would now be the minimum output (in horsepower) of an engine priced at $3,000?

(1) 500
(2) 400
(3) 300
(4) It is impossible to produce an engine of this kind.

Since the problem states that engines can no longer be produced using technology C, we can ignore this technology area and relate only to the other technology areas (the shaded regions in figure III). The starting point of this question is an engine whose price is $3,000. The vertical axis represents engine prices, and we will therefore begin with the vertical axis, from the point that represents a price of $3,000. The further we move to the right of this point, the greater the output. Thus, if we draw a horizontal line from the point on the vertical axis that represents a price of $3,000 (see figure III), the first point of contact with one of the technology areas will represent the **lowest** possible output for an engine priced at $3,000. The first point of contact with one of the shaded technology areas is with technology area D. This point lies on the vertical line representing 500 horsepower on the horizontal axis, which at present is the minimum output for an engine priced at $3,000. Therefore, the correct response is (1).

Figure III



4. A certain company is not allowed to produce engines with an output of over 550 horsepower. Which technologies can the company use to manufacture its engines?

(1) C only
(2) B and C only
(3) C and D only
(4) B, C and D only

The first step is to find the point on the horizontal axis that represents an output of 550 horsepower. Draw a vertical line from this point up the entire height of the graph (see figure IV). All engines to the right of this line have an output of **over** 550 horsepower, and all engines to the left of this line have an output of **less** than 550 horsepower. The company referred to in the question is only allowed to manufacture engines whose output is less than 550 horsepower. It can therefore use only those technologies whose areas, or part of whose areas, are to the **left** of the line. To the left of the line that we have drawn we find the entire area of technology C, half the area of technology B, and part of the area of technology D. Thus, the company can use technologies B, C and D for manufacturing engines with an output of less than 550 horsepower, and the correct response is (4).

Figure IV

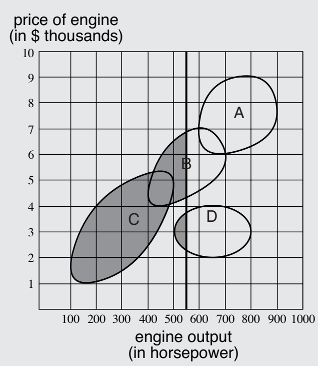


TABLE COMPREHENSION

INSTRUCTIONS:

Study the table below and answer the questions that follow.

EXPLANATION OF THE TABLE:

The table below contains data on 10 major companies belonging to different industries. The companies' names are designated by letters A through J, and appear in the first column of the table.

For each company, the table shows the industry to which it belongs, sales volume, profits, asset value, and number of workers.

For example, Company E deals in electronics, employs 400,000 workers, and its asset value is $90 million. Company E 's sales volume totaled $70 billion this year, and its profits amounted to $6,000 million.

The table also contains data on the percentage of change in sales and profits compared with last year.

An example of how to calculate percentage of change: If a certain company's sales volume totaled $40 billion last year, and this year the volume increased to $50 billion, then the percent of change compared to last year is 25%.

		Sales		Profits			
Name of company	Industry	Sales (in $ billions)	Percentage of change compared to last year	Profits (in $ millions)	Percentage of change compared to last year	Asset value (in $ millions)	Number of workers (in thousands)
A	Automobile	125	-1.5	-2,000	-150	180	750
B	Oil	110	25	6,500	0	100	150
C	Oil	105	22	5,000	40	390	100
D	Automobile	100	1.5	900	-80	180	350
E	Electronics	70	9	6,000	60	90	400
F	Automobile	65	7	3,000	15	55	100
G	Metals	60	25	1,000	-20	not given	400
H	Oil	60	20	3,000	-15	60	120
I	Oil	55	15	2,000	7	40	70
J	Electronics	50	6	4,500	10	150	300

Note: In answering each question, disregard the data appearing in the other questions.

QUESTIONS AND EXPLANATIONS

1. Which of the companies in the automobile industry has the **lowest** asset value?

(1) A
(2) D
(3) F
(4) A and D

This question requires you to read data. You have to locate the place in the table that shows the industry to which the company belongs and the place in the table that indicates its asset value. You then have to compare the asset value of all of the companies in the automobile industry, and find the lowest value. The second column from the left lists the industry of each company. It shows that companies A, D and F are the only companies in the automobile industry. Examining the asset value (second column from the right) of each of these companies, we see that the asset value of Company A is $180 million, which is also the asset value of Company D. The asset value of Company F is $55 million. Therefore, Company F has the lowest asset value of the companies in the automobile industry, and the correct response is (3).

2. Assuming that profits are divided equally among all the workers in a company, which of the following companies shows the **greatest** profit per individual worker?

(1) H
(2) B
(3) C
(4) F

The amount of profit per individual worker is not specified in the table but can be calculated from the information that does appear in it. The table shows the profit and number of workers of each company. The profit per individual worker of a particular company is the total profit of that company divided by the number of workers.

The profits of each company are given in millions of dollars, and the number of workers in thousands. Therefore, we can compare companies by relating to the numbers appearing in the table, and present the profit per worker as follows:

H	B	C	F
$\frac{3,000}{120}$	$\frac{6,500}{150}$	$\frac{5,000}{100}$	$\frac{3,000}{100}$

Comparing Companies F and H, we see that the same profit (3,000) is divided among fewer workers in Company F ($100 < 120$), and therefore the profit per worker is greater in Company F.
Comparing Companies F and C, we see that they have the same number of workers (100), but Company C has greater total profits ($5,000 > 3,000$), and therefore the profit per worker is greater for Company C.

Companies B and C are different both in terms of number of workers and in terms of total profits, and are therefore more difficult to compare. The number of workers in Company B is 1.5 times the number of workers in Company C (150 vs. 100); if the total profit of Company B were also 1.5 times greater than the profit of Company C, the profit per worker would be the same for both companies, that is, had the profit been $5{,}000 \cdot 1.5 = 7{,}500$; but the total profit of Company B is smaller than this amount ($6{,}500 < 7{,}500$). Thus, the profit per worker of Company B is smaller than the profit per worker in Company C. Hence, Company C has the greatest profit per individual worker, and the correct response is (3).

Another way of comparing Companies B and C:
In Company C, the profit per individual worker is 50 $\left(\frac{5{,}000}{100} = 50\right)$
while in Company B it is less than 50 $\left(\frac{6{,}500}{150} < 50\right)$ Therefore, the profit per individual worker in Company C is greater.

Another way of comparing the fractions that represent the profit per worker in each of the companies is by reducing them and converting them to a common denominator.

3. What was Company G's sales volume last year (in $ billions)?

(1) 48
(2) 50
(3) 64
(4) 76

This information does not appear in the table, but can be calculated using this year's sales volume (third column from the left) and the percent of change compared with last year (fourth column from the left). The table shows that Company G's sales this year amounted to $60 billion and that its sales increased by 25% over last year. In other words, its sales volume last year is a value which, if 25% is added to it, gives 60. This can be expressed in the following equation (where x is last year's sales volume): $x + \frac{25}{100} \cdot x = 60$.
Simplifying the equation: $\frac{125}{100} \cdot x = 60$
Solving for x: $x = 60 \cdot \frac{100}{125}$
Reducing the numerator and denominator of the fraction by 25: $x = 60 \cdot \frac{4}{5} = 48$

In other words, last year's sales volume was $48 billion, and the correct response is (1).

4. The sales of how many companies totaled over $100 billion last year?

(1) 1
(2) 2
(3) 3
(4) 4

This question also deals with last year's sales volume; thus, here too, we have to use the information provided about this year's sales volume and the percent of change compared with last year. We have to find companies whose sales last year amounted to more than $100 billion, but to do so it is not necessary to calculate each company's exact sales volume for last year. It is sufficient to know whether the sales volume was greater or smaller than this amount.
Companies E through J can easily be eliminated: Total sales for each of them this year amounted to less than $100 billion, and each had a positive percent of change compared to last year. In other words, this year's sales were greater than last year's, and therefore their volume of sales last year was obviously under $100 billion.

We will examine Companies A through D in greater detail. Company D's sales totaled $100 billion this year. Since its sales have increased over last year's sales (the percent of change was positive), last year's sales clearly totaled less than $100 billion.
Company C's sales this year amounted to $105 billion. If its sales last year had been over $100 billion, the change in sales would have been less than $5 billion, meaning the percent of change would have been less than 5% (since $5 billion is 5% of $100 billion). But since the change in sales was 22%, its sales last year clearly totaled less than $100 billion. We can eliminate Company B in the same way: Company B's sales totaled $110 billion this year. Had its sales last year amounted to more than $100 billion, the percent of change in its sales would have been less than 10%. However, it showed a 25% change in sales, and therefore last year's sales totaled less than $100 billion.

Company A's sales this year totaled $125 billion. Its sales decreased by 1.5% compared with last year. It follows that Company A's sales last year were over $125 billion. Thus, Company A is the only company whose sales last year totaled over $100 billion, and the correct response is (1).

QUANTITATIVE COMPARISONS

These questions require you to compare two quantities and determine which, if either, is greater. Sometimes, in addition to the two quantities, further information is given that is essential for answering the question. As in the sub-section on Questions and Problems, here too the questions cover a variety of areas: algebra, geometry (angle size, area and so on), calculating combinatorics and probabilities, and the like. As you will see in the wording of the instructions for the quantitative comparison questions, the response key is a uniform one for all of the questions in this sub-section, and it appears at the beginning of the sub-section instead of separately for each question.

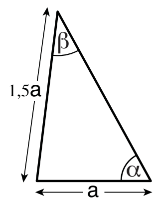
INSTRUCTIONS:

The following questions consist of pairs of quantities. In each question, one quantity appears in column A and a second quantity appears in column B. In the third column, additional information about the quantities in columns A and B is sometimes provided. **This information may be essential for answering the question.** Compare the two quantities, using the additional information (if provided), to determine which one of the following is true:

(1) the quantity in column A is greater
(2) the quantity in column B is greater
(3) the two quantities are equal
(4) there is not enough information to determine the relationship between the two quantities

For each question, mark the number of the answer you have chosen in the appropriate place on the answer sheet.

EXAMPLES AND EXPLANATIONS:

	Column A	Column B	Additional Information
1.	α	β	

To solve the question, you must analyze the additional information. The figure shows that angle α is opposite the side whose length is 1.5a, and angle β is opposite the side whose length is a. In any triangle, if side 1 is longer than side 2, the angle opposite side 1 is larger than the angle opposite side 2. Therefore, angle α, which is opposite the side whose length is 1.5a, is larger than angle β, which is opposite the side whose length is a, and the correct response is (1).

	Column A	Column B	Additional Information
2.	a	$\frac{a^3 - ab^2}{(a-b)\cdot(a+b)}$	$a \neq \pm b$

The additional information in this question prevents a situation whereby the denominator of the expression in Column B equals 0 (as this would create an undefined expression), but it does not directly help solve the question. For an easy comparison of the two expressions, the expression in Column B must be simplified.

Numerator: Factor out a: $(a^3 - ab^2) = a(a^2 - b^2)$.

Denominator: According to the contracted multiplication formula, $(a+b)\cdot(a-b) = a^2 - b^2$

The expression in Column B can be presented as $\frac{a\cdot(a^2-b^2)}{a^2-b^2}$.

Reduce the resulting fraction by dividing both numerator and denominator by $(a^2 - b^2)$. This is possible since $a \neq \pm b$, and therefore $(a^2 - b^2) \neq 0$: $\frac{a\cdot\cancel{(a^2-b^2)}}{\cancel{a^2-b^2}}$. The expression is equal to a.

Thus, the quantity in Column B is equal to the quantity in Column A, and the correct response is (3).

	Column A	Column B	Additional Information
3.	The average of: 0.03 , $\left(\frac{1}{2}\right)^a$, $\frac{2}{5}$	0.4	a is a positive integer.

The value of the expression in Column A depends on the size of a. To make it possible to compare the two quantities, we must look for the extreme cases, in which the expression takes its maximum or minimum value. Based on the additional information, a is a positive integer, i.e., its minimum value is 1.

The fraction $\left(\frac{1}{2}\right)^a$ becomes progressively smaller as the value of a becomes larger. Therefore, the expression in Column A has its **maximum** value when $a = 1$. We will calculate the maximum value of the average in Column A (where $a = 1$), and compare it to the expression in Column B:

Substituting $a = 1$ in column A, we obtain 0.03 , $\left(\frac{1}{2}\right)$, $\frac{2}{5}$.

Expressing these terms as fractions with a common denominator: $\frac{3}{100}$, $\frac{50}{100}$, $\frac{40}{100}$

Calculating the average: $\left(\frac{3}{100} + \frac{50}{100} + \frac{40}{100}\right) : 3 = \frac{93}{100} : 3 = \frac{93}{100} \cdot \frac{1}{3} = \frac{31}{100}$

The average that is obtained $\left(\frac{31}{100}\right)$ is smaller than the quantity in Column B ($\frac{40}{100}$). This is the maximum possible average in Column A. Therefore, for any value of a, the quantity in Column B is greater than the quantity in Column A, and the correct response is (2).

	Column A	Column B	Additional Information
4.	The sum to be paid for a product with starting price C, to which 15% VAT is added, followed by a 20% discount on the entire sum	The sum to be paid for a product with starting price C, after a 5% discount	$0 < C$

To compare the quantities in the two columns, each must be represented as an algebraic expression.

In order to calculate the quantity in Column A, add 15% to C: $C + \frac{15}{100}C = \frac{100C + 15C}{100} = \frac{115}{100}C$.

Subtract 20% from the resulting value:

$$\frac{115}{100}C - \frac{20}{100}\cdot\frac{115}{100}C = \frac{115}{100}C - \frac{1}{5}\cdot\frac{23}{20}C = \frac{115}{100}C - \frac{23}{100}C = \frac{92}{100}C$$

In order to calculate the quantity in Column B, subtract 5% from C: $C - \frac{5}{100}C = \frac{95}{100}C$.

Since we are provided with the additional information $0 < C$, the quantity in Column B is greater than the quantity in Column A, and the correct response is (2).

	Column A	Column B	Additional Information
5.	Twice the area of triangle ABC	The area of the circle circumscribing triangle ABC	ABC is a right triangle.

Drawing a diagram is helpful for solving this question:
It is known that any inscribed angle lying on a diameter of a circle equals 90°, and vice versa, any inscribed 90° angle lies on a diameter of a circle. Therefore, in a right triangle inscribed in a circle (triangle ABC), the hypotenuse (AC) is the diameter of the circle. We can see in the figure that if we double the triangle (triangle ADC is the mirror image of triangle ABC), the sum of the areas of the two triangles is smaller than the area of the circle. In other words, the quantity in Column B is greater than the quantity in Column A, and the correct response is (2).